A "One Line" Proof of the Slutsky Equation

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One focus of the usual classroom discussion of consumer theory is the demonstration that the individual consumer's reaction to a change in the market price of a commodity can be usefully broken down into vectors of substitution effects and income effects. The Slutsky equation relating the price effect to the substitution and income effects can be simply motivated by J. R. Hicks' graphical presentation, p. 31, but the usual proof (see Paul Samuelson) is very tedious and nonintuitive. If the instructor includes a discussion of the expenditure function in his curriculum, however, he has available a concise, intuitively appealing proof of the Slutsky equation.[1]

Suppose a consumer with income y faces a vector of commodity prices p. His Marshallian demand curve for commodity j is given by $x_j = D^j(y, p)$. The minimum expenditure necessary for the consumer to achieve any utility level u is given by his expenditure function, $y = m(u, p)$ (here y is in units of the jth good). His Hicksian income-compensated demand for commodity j is represented $x_j = h^j(u, p)$; if m is differentiable, we have the well-known result that

$$h^j(u, p) = \frac{\partial m(u, p)}{\partial p_j} \tag{1}$$

By the way the functions are defined, we have the identity

$$h^j(u, p) \equiv D^j(m[u, p], p) \tag{2}$$

Taking derivatives with respect to the price p_i of some commodity i yields, by the composite function rule:

$$\begin{aligned} \frac{\partial h^j(u, p)}{\partial p_i} &= \frac{\partial D^j(y, p)}{\partial y} \cdot \frac{\partial m(u, p)}{\partial p_i} \\ &\quad + \frac{\partial D^j(y, p)}{\partial p_i} \end{aligned} \tag{3}$$

where $y = m(u, p)$. Using (1) and rearranging terms gives us the Slutsky equation:

$$\frac{\partial D^j(y, p)}{\partial p_i} = \frac{\partial h^j(u, p)}{\partial p_i} - x_i \frac{\partial D^j(y, p)}{\partial y} \tag{4}$$

noting again that $y = m(u, p)$.

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[1] The expenditure function has been analyzed by L. McKenzie, S. Karlin, and D. McFadden and S. G. Winter, Jr.