



## A "One Line" Proof of the Slutsky Equation

Philip J. Cook

*The American Economic Review*, Vol. 62, No. 1/2 (1972), 139.

Stable URL:

<http://links.jstor.org/sici?sici=0002-8282%281972%2962%3A1%2F2%3C139%3AA%22LPOT%3E2.0.CO%3B2-A>

*The American Economic Review* is currently published by American Economic Association.

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://uk.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://uk.jstor.org/journals/aea.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

# A "One Line" Proof of the Slutsky Equation

By PHILIP J. COOK\*

One focus of the usual classroom discussion of consumer theory is the demonstration that the individual consumer's reaction to a change in the market price of a commodity can be usefully broken down into vectors of substitution effects and income effects. The Slutsky equation relating the price effect to the substitution and income effects can be simply motivated by J. R. Hicks' graphical presentation, p. 31, but the usual proof (see Paul Samuelson) is very tedious and non-intuitive. If the instructor includes a discussion of the expenditure function in his curriculum, however, he has available a concise, intuitively appealing proof of the Slutsky equation.<sup>1</sup>

Suppose a consumer with income  $y$  faces a vector of commodity prices  $p$ . His Marshallian demand curve for commodity  $j$  is given by  $x_j = D^j(y, p)$ . The minimum expenditure necessary for the consumer to achieve any utility level  $u$  is given by his expenditure function,  $y = m(u, p)$  (here  $y$  is in units of the  $j$ th good). His Hicksian income-compensated demand for commodity  $j$  is represented  $x_j = h^j(u, p)$ ; if  $m$  is differentiable, we have the well-known result that

$$(1) \quad h^j(u, p) = \frac{\partial m(u, p)}{\partial p_j}$$

By the way the functions are defined, we

\* Graduate student, University of California, Berkeley.

<sup>1</sup> The expenditure function has been analyzed by L. McKenzie, S. Karlin, and D. McFadden and S. G. Winter, Jr.

have the identity

$$(2) \quad h^j(u, p) \equiv D^j(m[u, p], p)$$

Taking derivatives with respect to the price  $p_i$  of some commodity  $i$  yields, by the composite function rule:

$$(3) \quad \frac{\partial h^j(u, p)}{\partial p_i} = \frac{\partial D^j(y, p)}{\partial y} \cdot \frac{\partial m(u, p)}{\partial p_i} + \frac{\partial D^j(y, p)}{\partial p_i}$$

where  $y = m(u, p)$ . Using (1) and rearranging terms gives us the Slutsky equation:

$$(4) \quad \frac{\partial D^j(y, p)}{\partial p_i} = \frac{\partial h^j(u, p)}{\partial p_i} - x_i \frac{\partial D^j(y, p)}{\partial y}$$

noting again that  $y = m(u, p)$ .

## REFERENCES

- J. R. Hicks, *Value and Capital*, 2d ed., London 1946.
- S. Karlin, *Mathematical Methods and Theory in Games, Programming, and Economics*, vol. 1, Reading, Mass. 1959.
- D. McFadden and S. G. Winter, Jr., *Theory of Resource Allocation and Prices*, unpublished manuscript, Univ. California, Berkeley 1969.
- L. McKenzie, "Demand Theory Without a Utility Index," *Rev. Econ. Stud.*, June 1957, 24, 185-89.
- P. A. Samuelson, *Foundations of Economic Analysis*, New York 1967.