CHAPTER 5
UNCERTAINTY AND CONSUMER BEHAVIOR

QUESTIONS FOR REVIEW

1. What does it mean to say that a person is risk averse? Why are some people likely to be risk averse, while others are risk lovers?

A risk-averse person has a diminishing marginal utility of income and prefers a certain income to a gamble with the same expected income. A risk lover has an increasing marginal utility of income and prefers an uncertain income to a certain income. The economic explanation of whether an individual is risk averse or risk loving depends on the shape of the individual's utility function for wealth. Also, a person's risk aversion (or risk loving) depends on the nature of the risk involved and on the person's income.

2. Why is the variance a better measure of variability than the range?

Range is the difference between the highest possible outcome and the lowest possible outcome. Range does not indicate the probabilities of observing these high or low outcomes. Variance weighs the difference of each outcome from the mean outcome by its probability and, thus, is a more useful measure of variability than the range.

3. George has $5,000 to invest in a mutual fund. The expected return on mutual fund A is 15% and the expected return on mutual fund B is 10%. Should George pick mutual fund A or fund B?

George's decision will depend not only on the expected return for each fund, but also on the variability in the expected return on each fund, and on George's preferences. For example, if fund A has a higher standard deviation than fund B, and George is risk averse, then he may prefer fund B even though it has a lower expected return. If George is not particularly risk averse he may choose fund A even if it subject to more variability in its expected return.

4. What does it mean for consumers to maximize expected utility? Can you think of a case where a person might not maximize expected utility?

The expected utility is the sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur. To maximize expected utility means that the individual chooses the option that yields the highest average utility, where average utility is a probability-weighted sum of all utilities. This theory requires that the consumer knows the probability of every outcome. At times, consumers either do not know the relevant probabilities or have difficulty in evaluating low-probability, high-payoff events. In some cases, consumers cannot assign a utility level to these high-payoff events, such as when the payoff is the loss of the consumer's life.

5. Why do people often want to insure fully against uncertain situations even when the premium paid exceeds the expected value of the loss being insured against?

If the cost of insurance is equal to the expected loss, (i.e., if the insurance is actuarially fair), risk-averse individuals will fully insure against monetary loss. The insurance premium assures the individual of having the same income regardless of whether or not a loss occurs. Because the insurance is actuarially fair, this certain income is equal to the expected income if the individual takes the risky option of not purchasing insurance. This guarantee of the same income, whatever the outcome, generates more utility for a risk-averse person than the average utility of a high income when there was no loss and the utility of a low income with a loss (i.e., because of risk aversion, \( E[U(x)] \leq U(E[x]) \)).
6. Why is an insurance company likely to behave as if it is risk neutral even if its managers are risk-averse individuals?

Most large companies have opportunities for diversifying risk. Managers acting for the owners of a company choose a portfolio of independent, profitable projects at different levels of risk. Of course, shareholders may diversify their risk by investing in several projects in the same way that the insurance company itself diversifies risk by insuring many people. By operating on a sufficiently large scale, insurance companies can assure themselves that over many outcomes the total premiums paid to the company will be equal to the total amount of money paid out to compensate the losses of the insured. Thus, the insurance company behaves as if it is risk neutral, while the managers, as individuals, might be risk averse.

7. When is it worth paying to obtain more information to reduce uncertainty?

Individuals are willing to pay more for information when the utility of the choice with more information, including the cost of gathering the information, is greater than the expected utility of the choice without the information.

8. How does the diversification of an investor’s portfolio avoid risk?

An investor reduces risk by investing in many inversely related assets. For example, a mutual fund is a portfolio of stocks of independent companies. If the variance of the return on one company’s stock is inversely related to the variance of the return on another company’s stock, a portfolio of both stocks will have a lower variance than either stock held separately. As the number of stocks increases, the variance in the rate of return on the portfolio as a whole decreases. While there is less risk in a portfolio of stocks, risk is not eliminated altogether; there is still some market risk in holding such a portfolio, compared to a low-risk asset, such as a U.S. government savings bond.

9. Why do some investors put a large portion of their portfolios into risky assets, while others invest largely in risk-free alternatives? (Hint: Do the two investors receive exactly the same return on average? Why?)

In a market for risky assets, where investors are risk averse, investors demand a higher return on investments that have a higher level of risk (a higher variance in returns). Although some individuals are willing to accept a higher level of risk in exchange for a higher rate of return, this does not mean that these individuals are less risk averse. On the contrary, they will not invest in risky assets unless they are compensated for the increased risk.

10. What is an endowment effect? Give an example of such an effect.

An endowment effect exists if an individual places a greater value on an item that is in her possession as compared to the value she places on the same item when it is not in her possession. For example, many people would refuse to pay $5 for a simple coffee mug but would also refuse to sell a simple coffee mug they won in a contest for the same price even though they got it for free.

11. Jennifer is shopping and sees an attractive shirt. However, the price of $50 is more than she is willing to pay. A few weeks later she finds the same shirt on sale for $25, and buys it. When a friend offers Jennifer $50 for the shirt, she refuses to sell it. Explain Jennifer’s behavior.

To help explain Jennifer’s behavior, we need to look at the reference point from which she is making the decision. In the first instance, she does not own the shirt so she is not willing to pay the $50 to buy the shirt. In the second instance, she will not accept $50 for the shirt from her friend because her reference point has changed. Once she owns the shirt, she changed the amount by which she valued the shirt. Individuals often value goods more when they own them than when they do not.
EXERCISES

1. Consider a lottery with three possible outcomes: $125 will be received with probability .2, $100 with probability .3, and $50 with probability .5.

   a. What is the expected value of the lottery?
      
      The expected value, $EV$, of the lottery is equal to the sum of the returns weighted by their probabilities:
      
      $$ EV = (0.2)(125) + (0.3)(100) + (0.5)(50) = 80. $$

   b. What is the variance of the outcomes of the lottery?
      
      The variance, $\sigma^2$, is the sum of the squared deviations from the mean, $80$, weighted by their probabilities:
      
      $$ \sigma^2 = (0.2)(125 - 80)^2 + (0.3)(100 - 80)^2 + (0.5)(50 - 80)^2 = 975. $$

   c. What would a risk-neutral person pay to play the lottery?
      
      A risk-neutral person would pay the expected value of the lottery: $80.
2. Suppose you have invested in a new computer company whose profitability depends on (1) whether the U.S. Congress passes a tariff that raises the cost of Japanese computers and (2) whether the U.S. economy grows slowly or quickly. What are the four mutually exclusive states of the world that you should be concerned about?

The four mutually exclusive states may be represented as:

<table>
<thead>
<tr>
<th>Slow growth rate</th>
<th>Fast growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1:</td>
<td>State 3:</td>
</tr>
<tr>
<td>Slow growth with tariff</td>
<td>Fast growth with tariff</td>
</tr>
<tr>
<td>State 2:</td>
<td>State 4:</td>
</tr>
<tr>
<td>Slow growth without tariff</td>
<td>Fast growth without tariff</td>
</tr>
</tbody>
</table>

3. Richard is deciding whether to buy a state lottery ticket. Each ticket costs $1, and the probability of the following winning payoffs is given as follows:

<table>
<thead>
<tr>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>$0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>$1.00</td>
</tr>
<tr>
<td>0.20</td>
<td>$2.00</td>
</tr>
<tr>
<td>0.05</td>
<td>$7.50</td>
</tr>
</tbody>
</table>

a. What is the expected value of Richard’s payoff if he buys a lottery ticket? What is the variance?

The expected value of the lottery is equal to the sum of the returns weighted by their probabilities:

\[ EV = (0.5)(0) + (0.25)(1) + (0.2)(2) + (0.05)(7.5) = 1.025 \]

The variance is the sum of the squared deviation from the mean, weighted by their probabilities:

\[ \sigma^2 = (0.5)(0 - 1.025)^2 + (0.25)(1 - 1.025)^2 + (0.2)(2 - 1.025)^2 + (0.05)(7.5 - 1.025)^2, \text{ or} \]

\[ \sigma^2 = 2.812. \]

b. Richard’s nickname is “No-risk Rick.” He is an extremely risk-averse individual. Would he buy the ticket?

An extremely risk-averse individual will probably not buy the ticket, even though the expected outcome is higher than the price, $1.025 > $1.00. The difference in the expected return is not enough to compensate Rick for the risk. For example, if his wealth is $10 and he buys a $1.00 ticket, he would have $9.00, $10.00, $11.00, and $16.50, respectively, under the four possible outcomes. Let us assume that his utility function is \( U = W^{0.5} \), where \( W \) is his wealth. Then his expected utility is:

\[ EU = (0.5)(9^{0.5}) + (0.25)(1^{0.5}) + (0.2)(1^{0.5}) + (0.05)(6.5^{0.5}) = 3.157. \]

This is less than 3.162, which is the utility associated with not buying the ticket \( (U(10) = 10^{0.5} = 3.162) \). He would prefer the sure thing, i.e., $10.

c. Suppose Richard was offered insurance against losing any money. If he buys 1,000 lottery tickets, how much would he be willing to pay to insure his gamble?

If Richard buys 1,000 tickets, it is likely that he will have $1,025 minus the $1,000 he paid, or $25. He would not buy any insurance, as the expected return, $1,025, is greater than the cost, $1,000. He has insured himself by buying a large number of tickets. Given that Richard is risk averse though, he may still want to buy insurance. The amount he would be willing to pay is equal to the risk premium, which is the
amount of money that Richard would pay to avoid the risk. See figure 5.4 in the text. To calculate the risk premium, you need to know the utility function. If the utility function is \( U = W^{0.5} \), then his expected utility from the 1,000 lottery tickets is

\[
EU = (0.5)(0.0^{0.5}) + (0.25)(1000^{0.5}) + (0.2)(2000^{0.5}) + (0.05)(7500^{0.5}) = 21.18.
\]

This is less than the utility he would get from keeping his $1000 which is \( U=1000^{0.5}=31.62 \). To find the risk premium, find the level of income that would guarantee him a utility of 21.18, which is $448.59. This means he would pay $1000-$448.59=$551.41 to insure his gamble.

d. In the long run, given the price of the lottery ticket and the probability/return table, what do you think the state would do about the lottery?

In the long run, the state lottery will be bankrupt! Given the price of the ticket and the probabilities, the lottery is a money loser. The state must either raise the price of a ticket or lower the probability of positive payoffs.

4. Suppose an investor is concerned about a business choice in which there are three prospects, whose probability and returns are given below:

<table>
<thead>
<tr>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>$100</td>
</tr>
<tr>
<td>0.3</td>
<td>30</td>
</tr>
<tr>
<td>0.3</td>
<td>-30</td>
</tr>
</tbody>
</table>

What is the expected value of the uncertain investment? What is the variance?

The expected value of the return on this investment is

\[
EV = (0.4)(100) + (0.3)(30) + (0.3)(-30) = $40.
\]

The variance is

\[
\sigma^2 = (0.4)(100 - 40)^2 + (0.3)(30 - 40)^2 + (0.3)(-30 - 40)^2 = $2,940.
\]

5. You are an insurance agent who has to write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich condiment industry. The sandwich industry will pay top dollar to whoever invents such a mayonnaise substitute first.

Sam’s SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>.999</td>
<td>-$1,000,000</td>
</tr>
<tr>
<td>.001</td>
<td>$1,000,000,000</td>
</tr>
</tbody>
</table>

(a) What is the expected return of his project? What is the variance?

The expected return, \( ER \), of the investment is

\[
ER = (0.999)(-1,000,000) + (0.001)(1,000,000,000) = $1,000.
\]

The variance is

\[
\sigma^2 = (0.999)(-1,000,000 - 1,000)^2 + (0.001)(1,000,000,000 - 1,000)^2, \text{ or } \sigma^2 = 1,000,998,999,000,000.
\]
b. What is the most Sam is willing to pay for insurance? Assume Sam is risk neutral.

Because Sam is risk neutral and because the expected outcome is $1,000, Sam is unwilling to buy insurance.

c. Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1,000 for the insurance. Assume that Sam tells you SCAM is only six months away from perfecting its mayonnaise substitute and that you know what you know about the Japanese. Would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?

The entry of the Japanese lowers Sam’s probability of a high payoff. For example, assume that the probability of the billion dollar payoff is lowered to zero. Then the expected outcome is:

\[(1.0)(-1,000,000) + (0.0)(1,000,000,000,000) = -1,000,000.\]

Therefore, you should raise the policy premium substantially. But Sam, not knowing about the Japanese entry, will continue to refuse your offers to insure his losses.

6. Suppose that Natasha’s utility function is given by \(u(I) = \sqrt{10I}\), where \(I\) represents annual income in thousands of dollars.


Natasha is risk averse. To show this, assume that she has $10,000 and is offered a gamble of a $1,000 gain with 50 percent probability and a $1,000 loss with 50 percent probability. Her utility of $10,000 is 10, \(u(I) = \sqrt{10*10} = 10\). Her expected utility is:

\[EU = (0.5)(90^{0.5}) + (0.5)(110^{0.5}) = 9.987 < 10.\]

She would avoid the gamble. If she were risk neutral, she would be indifferent between the $10,000 and the gamble; whereas, if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by noting that the second derivative is negative, implying diminishing marginal utility.

b. Suppose that Natasha is currently earning an income of $40,000 \((I = 40)\) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .6 probability of earning $44,000, and a .4 probability of earning $33,000. Should she take the new job?

The utility of her current salary is \(400^{0.5}\), which is 20. The expected utility of the new job is

\[EU = (0.6)(440^{0.5}) + (0.4)(330^{0.5}) = 19.85,\]

which is less than 20. Therefore, she should not take the job.

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)

Assuming that she takes the new job, Natasha would be willing to pay a risk premium equal to the difference between $40,000 and the utility of the gamble so as to ensure that she obtains a level of utility equal to 20. We know the utility of the gamble is equal to 19.85. Substituting into her utility function we have, \(19.85 = (10I)^{0.5}\), and solving for \(I\) we find the income associated with the gamble to be $39,410. Thus, Natasha would be willing to pay for insurance equal to the risk premium, $40,000 - $39,410 = $590.
7. Suppose that two investments have the same three payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

<table>
<thead>
<tr>
<th>Payoff</th>
<th>Probabilities for Investment A</th>
<th>Probabilities for Investment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$300</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>$250</td>
<td>0.80</td>
<td>0.40</td>
</tr>
<tr>
<td>$200</td>
<td>0.10</td>
<td>0.30</td>
</tr>
</tbody>
</table>

a. Find the expected return and standard deviation of each investment.

The expected value of the return on investment A is

\[ EV_A = (0.1)(300) + (0.8)(250) + (0.1)(200) = 250. \]

The variance on investment A is

\[ \sigma^2_A = (0.1)(300 - 250)^2 + (0.8)(250 - 250)^2 + (0.1)(200 - 250)^2 = 500. \]

The expected value of the return on investment B is

\[ EV_B = (0.3)(300) + (0.4)(250) + (0.3)(200) = 250. \]

The variance on investment B is

\[ \sigma^2_B = (0.3)(300 - 250)^2 + (0.4)(250 - 250)^2 + (0.3)(200 - 250)^2 = 1500. \]

b. Jill has the utility function \( U = 5I \), where I denotes the payoff. Which investment will she choose?

Jill’s expected utility from investment A is

\[ EU_A = 0.1(5\times300) + 0.8(5\times250) + 0.1(5\times200) = 1250. \]

Jill’s expected utility from investment B is

\[ EU_B = 0.3(5\times300) + 0.4(5\times250) + 0.3(5\times200) = 1250. \]

Since both investments give Jill the same expected utility she will be indifferent between the two.

c. Ken has the utility function \( U = \sqrt{5I} \). Which investment will he choose?

Ken’s expected utility from investment A is

\[ EU_A = 0.1(5\times300)^{0.5} + 0.8(5\times250)^{0.5} + 0.1(5\times200)^{0.5} = 35.32. \]

Ken’s expected utility from investment B is

\[ EU_B = 0.3(5\times300)^{0.5} + 0.4(5\times250)^{0.5} + 0.3(5\times200)^{0.5} = 35.25. \]

Ken will choose investment A since it has a higher expected utility. Notice that since Ken is risk averse, he will prefer the investment with less variability.

d. Laura has the utility function \( U = 5I^2 \). Which investment will she choose?

Laura’s expected utility from investment A is

\[ EU_A = 0.1(5\times300^2) + 0.8(5\times250^2) + 0.1(5\times200^2) = 315000. \]

Laura’s expected utility from investment B is

\[ EU_B = 0.3(5\times300^2) + 0.4(5\times250^2) + 0.3(5\times200^2) = 320000. \]

Laura will choose investment B since it has a higher expected utility.
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8. As the owner of a family farm whose wealth is $250,000, you must choose between sitting this season out and investing last year's earnings ($200,000) in a safe money market fund paying 5.0% or planting summer corn. Planting costs $200,000, with a six-month time to harvest. If there is rain, planting summer corn will yield $500,000 in revenues at harvest. If there is a drought, planting will yield $50,000 in revenues at harvest. As a third choice, you can purchase AgriCorp drought-resistant summer corn at a cost of $250,000 that will yield $500,000 in revenues at harvest if there is rain, and $350,000 in revenues at harvest if there is a drought. You are risk averse and your preferences for family wealth (W) are specified by the relationship \( U(W) = \sqrt{W} \). The probability of a summer drought is 0.30 and the probability of summer rain is 0.70. Which of the three options should you choose? Explain.

You need to calculate expected utility of wealth under the three options. Wealth is equal to the initial $250,000 plus whatever is earned on growing corn, or investing in the safe financial asset. Expected utility under the safe option allowing for the fact that your initial wealth is $250,000 is:

\[
E(U) = (250,000 + 200,000(1 + .05))^{0.5} = 678.23.
\]

Expected utility with regular corn, again including your initial wealth:

\[
E(U) = .7(250,000 + (500,000 - 200,000))^{0.5} + .3(250,000 + (50,000 - 200,000))^{0.5} = 519.13 + 94.87 = 614.
\]

Expected utility with drought-resistant corn, again including your initial wealth:

\[
E(U) = .7(250,000 + (500,000 - 250,000))^{0.5} + .3(250,000 + (350,000 - 250,000))^{0.5} = 494.975 + 177.482 = 672.46.
\]

You should choose the option with the highest expected utility, which is the safe option of not planting corn.

9. Draw a utility function over income \( u(I) \) that has the property that a man is a risk lover when his income is low but a risk averter when his income is high. Can you explain why such a utility function might reasonably describe a person’s preferences?

Consider an individual who needs a certain level of income, \( I^* \), in order to stay alive. An increase in income above \( I^* \) will have a diminishing marginal utility. Below \( I^* \), the individual will be a risk lover and will take unfair gambles in an effort to make large gains in income. Above \( I^* \), the individual will purchase insurance against losses.
10. A city is considering how much to spend monitoring parking meters. The following information is available to the city manager:

i. Hiring each meter-monitor costs $10,000 per year.

ii. With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to .25.

iii. With two monitors hired, the probability of getting a ticket is .5, with three monitors the probability is .75, and with four the probability is equal to 1.

iv. The current fine for overtime parking with two metering persons hired is $20.

a. Assume first that all drivers are risk-neutral. What parking fine would you levy and how many meter monitors would you hire (1, 2, 3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?

If drivers are risk neutral, their behavior is only influenced by the expected fine. With two meter-monitors, the probability of detection is 0.5 and the fine is $20. So, the expected fine is $10 = (0.5)($20). To maintain this expected fine, the city can hire one meter-monitor and increase the fine to $40, or hire three meter-monitors and decrease the fine to $13.33, or hire four meter-monitors and decrease the fine to $10.

If the only cost to be minimized is the cost of hiring meter-monitors, i.e., $10,000 per year, you as the city manager, should minimize the number of meter-monitors. Hire only one monitor and increase the fine to $40 to maintain the current level of deterrence.

b. Now assume that drivers are highly risk averse. How would your answer to (a) change?

If drivers are risk averse, their utility of a certain outcome is greater than their utility of an expected value equal to the certain outcome. They will avoid the possibility of paying a parking fine more than would risk-neutral drivers. Therefore, a fine of less than $40 will maintain the current level of deterrence.

c. (For discussion) What if drivers could insure themselves against the risk of parking fines? Would it make good public policy to permit such insurance?

Drivers engage in many forms of behavior to insure themselves against the risk of parking fines, such as parking blocks away from their destination in a non-metered spot or taking public transportation. A private insurance firm could offer an insurance
policy to pay fines if a ticket is received. Of course, the premium for such insurance would be based on each driver’s probability of receiving a parking ticket and on the opportunity cost of providing service. (Note: full insurance leads to moral hazard problems, to be discussed in Chapter 17.)

Public policy should attempt to maximize the difference between the benefits and costs to all parties. Private insurance may not be optimal, because of the increase in transactions costs. Instead, as the city manager, consider offering another form of insurance, e.g., the selling of parking stickers, and give tickets for inappropriately parked cars.
11. A moderately risk-averse investor has 50 percent of her portfolio invested in stocks and 50 percent invested in risk-free Treasury bills. Show how each of the following events will affect the investor's budget line and the proportion of stocks in her portfolio:

a. The standard deviation of the return on the stock market increases, but the expected return on the stock market remains the same.

From section 5.4, the equation for the budget line is

\[ R_p = \left( \frac{R_m - R_f}{\sigma_m} \right) \sigma_p + R_f, \]

where \( R_p \) is the expected return on the portfolio, \( R_m \) is the expected return on the risky asset, \( R_f \) is the expected return on the risk-free asset, \( \sigma_m \) is the standard deviation of the return on the risky asset, and \( \sigma_p \) is the standard deviation of the return on the portfolio.

The budget line shows the positive relationship between the return on the portfolio, \( R_p \), and the standard deviation of the return on the portfolio, \( \sigma_p \).

In this case, if the standard deviation of the return on the stock market increases, \( \sigma_m \), the slope of the budget line will change and the budget line will become flatter. At any given level of portfolio return, there is now a higher standard deviation associated with that level of return. You would expect the proportion of stocks in the portfolio to fall.

b. The expected return on the stock market increases, but the standard deviation of the stock market remains the same.

In this case, if the expected return on the stock market increases, \( R_m \), then the slope of the budget line will change and the budget line will become steeper. At any given level of standard deviation of return, \( \sigma_p \), there is now a higher level of return, \( R_p \). You would expect the proportion of stocks in the portfolio to rise.

c. The return on risk-free Treasury bills increases.

In this case there is an increase in \( R_f \). The budget line will pivot and shift, or in other words will shift up and become flatter. The proportion of stocks in the portfolio could go either way. On the one hand, Treasury bills now have a higher return and so are more attractive. On the other hand, the investor can now earn a higher return from each Treasury bill so could hold fewer Treasury bills and still maintain the same return in terms of the total flow or payment from the Treasury bills. In this second case, the investor may be willing to place more of his savings into the riskier asset. It will depend on the particular preferences of the investor, as well as the magnitude of the returns to the two assets. An analogy would be to consider what happens to savings when the interest rate increases. On the one hand it goes up because the return is higher, but on the other hand it can go down because a person can save less each period and still come out with the same accumulation of savings at some future date.