CHAPTER 13
GAME THEORY AND COMPETITIVE STRATEGY

REVIEW QUESTIONS

1. What is the difference between a cooperative and a noncooperative game? Give an example of each.

In a noncooperative game the players do not formally communicate in an effort to coordinate their actions. They are aware of one another's existence, but act independently. The primary difference between a cooperative and a noncooperative game is that a binding contract, i.e., an agreement between the parties to which both parties must adhere, is possible in the former, but not in the latter. An example of a cooperative game would be a formal cartel agreement, such as OPEC, or a joint venture. An example of a noncooperative game would be a race in research and development to obtain a patent.

2. What is a dominant strategy? Why is an equilibrium stable in dominant strategies?

A dominant strategy is one that is best no matter what action is taken by the other party to the game. When both players have dominant strategies, the outcome is stable because neither party has an incentive to change.

3. Explain the meaning of a Nash equilibrium. How does it differ from an equilibrium in dominant strategies?

A Nash equilibrium is an outcome where both players correctly believe that they are doing the best they can, *given the action of the other player*. A game is in equilibrium if neither player has an incentive to change his or her choice, unless there is a change by the other player. The key feature that distinguishes a Nash equilibrium from an equilibrium in dominant strategies is the *dependence* on the opponent's behavior. An equilibrium in dominant strategies results if each player has a best choice, regardless of the other player's choice. Every dominant strategy equilibrium is a Nash equilibrium but the reverse does not hold.

4. How does a Nash equilibrium differ from a game's maximin solution? In what situations is a maximin solution a more likely outcome than a Nash equilibrium?

A maximin strategy is one in which each player determines the worst outcome for each of the opponent's actions and chooses the option that maximizes the minimum gain that can be earned. Unlike the Nash equilibrium, the maximin solution does not require players to react to an opponent's choice. If no dominant strategy exists (in which case outcomes depend on the opponent's behavior), players can reduce the uncertainty inherent in relying on the opponent's rationality by conservatively following a maximin strategy. The maximin solution is more likely than the Nash solution in cases where there is a higher probability of irrational (non-optimizing) behavior.

5. What is a "tit-for-tat" strategy? Why is it a rational strategy for the infinitely repeated prisoners' dilemma?

A player following a "tit-for-tat" strategy will cooperate as long as his or her opponent is cooperating and will switch to a noncooperative strategy if their opponent switches strategies. When the competitors assume that they will be repeating their interaction in *every* future period, the long-term gains from cooperating will outweigh any short-term gains from not cooperating. Because the "tit-for-tat" strategy encourages cooperation in infinitely repeated games, it is rational.

6. Consider a game in which the prisoners' dilemma is repeated 10 times and both players are rational and fully informed. Is a tit-for-tat strategy optimal in this case? Under what conditions would such a strategy be optimal?

Since cooperation will unravel from the last period back to the first period, the "tit-for-tat" strategy is not optimal when there is a finite number of periods and both players anticipate the competitor's response in *every* period. Given that there is no response possible in the eleventh period for action in the tenth (and last) period, cooperation breaks down in the last period. Then, knowing that there is no cooperation in the last period, players should maximize their self-interest by not cooperating in the second-to-last period. This unraveling occurs because both players assume that the other player has considered *all* consequences in *all* periods. However, if there is some doubt about whether the opponent has fully anticipated the consequences of the "tit-for-tat" strategy in the final period, the game will not unravel and the "tit-for-tat" strategy can be optimal.

7. Suppose you and your competitor are playing the pricing game shown in Table 13.8. Both of you must announce your prices at the same time. Can you improve your outcome by promising your competitor that you will announce a high price?

If the game is to be played only a few times, there is little to gain. If you are Firm 1 and promise to announce a high price, Firm 2 will undercut you and you will end up with a payoff of -50. However, next period you will undercut too, and both firms will earn 10. If the game is played many times, there is a better chance that Firm 2 will realize that if it matches your high price, the long-term payoff of 50 each period is better than 100 at first and 10 thereafter.

8. What is meant by "first-mover advantage"? Give an example of a gaming situation with a first-mover advantage.

A "first-mover" advantage can occur in a game where the first player to act receives the highest payoff. The first-mover signals his or her choice to the opponent, and the opponent must choose a response, *given this signal*. The first-mover goes on the offensive and the second-mover responds defensively. In many recreational games, from chess to football, the first-mover has an advantage. In many markets, the first firm to introduce a product can set the standard for competitors to follow. In some cases, the standard-setting power of the first mover becomes so pervasive in the market that the brand name of the product becomes synonymous with the product, e.g., "Kleenex," the name of Kleenex-brand facial tissue, is used by many consumers to refer to facial tissue of any brand.

9. What is a "strategic move"? How can the development of a certain kind of reputation be a strategic move?

A strategic move involves a commitment to reduce one's options. The strategic move might not seem rational outside the context of the game in which it is played, but it is rational given the anticipated response of the other player. Random responses to an opponent's action may not appear to be rational, but developing a reputation of being unpredictable could lead to higher payoffs in the long run. Another example would be making a promise to give a discount to all previous consumers if you give a discount to one. Such a move makes the firm vulnerable, but the goal of such a strategic move is to signal to rivals that you *won't* be discounting price and hope that your rivals follow suit.

10. Can the threat of a price war deter entry by potential competitors? What actions might a firm take to make this threat credible?

Both the incumbent and the potential entrant know that a price war will leave their firms worse off. Normally, such a threat is not credible. Thus, the incumbent must make his or her threat of a price war believable by signaling to the potential entrant that a price war *will* result if entry occurs. One strategic move is to increase capacity, signaling a lower future price, and another is to engage in apparently irrational behavior. Both types of strategic behavior might deter entry, but for different reasons. While an increase in capacity reduces expected profits by reducing prices, irrational behavior reduces expected profits by increasing uncertainty, hence increasing the rate at which future profits must be discounted into the present.

11. A strategic move limits one's flexibility and yet gives one an advantage. Why? How might a strategic move give one an advantage in bargaining?

A strategic move influences conditional behavior by the opponent. If the game is well understood and the opponent's reaction can be predicted, a strategic move leaves the player better off. Economic transactions involve a bargain, whether implicit or explicit. In every bargain, we assume that both parties attempt to maximize their self-interest. Strategic moves by one player provide signals to which another player reacts. If a bargaining game is played only once (so no reputations are involved), the players might act strategically to maximize their payoffs. If bargaining is repeated, players might act strategically to establish reputations for expected negotiations.

12. Why is the winner's curse potentially a problem for a bidder in a common value auction but not in a private value auction?

The winner's curse states that "The winner of a common value auction is likely to be made worse off (than not winning) because the winner was overly optimistic and, as a consequence, bid more for the item than it was actually worth." In a private value auction, you are aware of your own reservation price, and will bid accordingly. Once the price has escalated above your reservation price, you will no longer bid. If you win, it is because the winning bid was below your reservation price. In a common value auction, you do not know the exact value of the good you are bidding on. The winner will tend to be the person who has most overestimated the value of the good, assuming that some bidders overestimate and some underestimate. If all bids are below the actual value, then there is no winner's curse.

EXERCISES

1. In many oligopolistic industries, the same firms compete over a long period of time, setting prices and observing each other's behavior repeatedly. Given that the number of repetitions is large, why don't collusive outcomes typically result?

If games are repeated indefinitely and all players know all payoffs, rational behavior will lead to apparently collusive outcomes, i.e., the same outcomes that would result if firms were actively colluding. All payoffs, however, might not be known by all players. Sometimes the payoffs of other firms can only be known by engaging in extensive (and costly) information exchanges or by making a move and observing rivals' responses. Also, successful collusion encourages entry. Perhaps the greatest problem in maintaining a collusive outcome is that changes in market conditions change the collusive price and quantity. The firms then have to repeatedly change their agreement on price and quantity, which is costly, and this increases the ability of one firm to cheat without being discovered.

2. Many industries are often plagued by overcapacity--firms simultaneously make major investments in capacity expansion, so total capacity far exceeds demand. This happens not only in industries in which demand is highly volatile and unpredictable, but also in industries in which demand is fairly stable. What factors lead to overcapacity? Explain each briefly.

In Chapter 12, we found that excess capacity may arise in industries with easy entry and differentiated products. In the monopolistic competition model, downward-sloping demand curves for each firm lead to output with average cost above minimum average cost. The difference between the resulting output and the output at minimum long-run average cost is defined as excess capacity. In this chapter, we saw that overcapacity could be used to deter new entry; that is, investments in capacity expansion could convince potential competitors that entry would be unprofitable. (Note that although threats of capacity expansion may deter entry, these threats must be *credible*.)

3. Two computer firms, A and B, are planning to market network systems for office information management. Each firm can develop either a fast, high-quality system (H), or a slower, low-quality system (L). Market research indicates that the resulting profits to each firm for the alternative strategies are given by the following payoff matrix:

		Firm B	
		H	***L***
Firm A	***H***	**50, 40**	**60, 45**
	L	**55, 55**	**15, 20**

a. If both firms make their decisions at the same time and follow *maximin* (low-risk) strategies, what will the outcome be?

With a maximin strategy, a firm determines the worst outcome for each option, then chooses the option that maximizes the payoff among the worst outcomes. If Firm A chooses *H*, the worst payoff would occur if Firm B chooses *H*: A's payoff would be 50. If Firm A chooses *L*, the worst payoff would occur if Firm B chooses *L*: A's payoff would be 15. With a maximin strategy, A therefore chooses *H*. If Firm B chooses *L*, the worst payoff would occur if Firm A chooses *L*: the payoff would be 20. If Firm B chooses *H*, the worst payoff, 40, would occur if Firm A chooses *H*. With a maximin strategy, B therefore chooses *H*. So under maximin, both A and B produce a high-quality system.

b. Suppose both firms try to maximize profits, but Firm A has a head start in planning, and can commit first. Now what will the outcome be? What will the outcome be if Firm B has a head start in planning and can commit first?

If Firm A can commit first, it will choose *H*, because it knows that Firm B will rationally choose *L*, since *L* gives a higher payoff to B (45 vs. 40). This gives Firm A a payoff of 60. If Firm A instead committed to L, B would choose H (55 vs. 20), giving A 55 instead of 60. If Firm B can commit first, it will choose *H*, because it knows that Firm A will rationally choose *L*, since *L* gives a higher payoff to A (55 vs. 50). This gives Firm B a payoff of 55.

c. Getting a head start costs money (you have to gear up a large engineering team). Now consider the *two-stage* game in which *first*, each firm decides how much money to spend to speed up its planning, and *second*, it announces which product (H or L) it will produce. Which firm will spend more to speed up its planning? How much will it spend? Should the other firm spend *anything* to speed up its planning? Explain.

In this game, there is an apparent advantage to being the first mover. If A moves first, its profit is 60. If it moves second, its profit is 55, a difference of 5. Thus, it would be willing to spend up to 5 for the option of announcing first. On the other hand, if B moves first, its profit is 55. If it moves second, its profit is 45, a difference of 10, and it thus would be willing to spend up to 10 for the option of announcing first.

Once Firm A realizes that Firm B is willing to spend something on the option of announcing first, then the value of the option decreases for Firm A, because if both firms were to invest both firms would choose to produce the high-quality system, which gives them both a lower payoff. Firm A should not spend money to speed up the introduction of its product in any case. If B goes first and chooses high then A can choose low and end up with 55 instead of the 50 it would get if it went high also. This is best for B also because if B goes high and A low, B ends up with 55 instead of 40, so even if it spends the 10 it is still ahead by 5. A should let B go first. Finally, note that even though B is better off if A goes first and chooses high (45 vs. 40), B is still best off if it goes first. Overall, it is worthwhile for B to spend the money and announce but it is not worthwhile for A to spend any money.

4. Two firms are in the chocolate market. Each can choose to go for the high end of the market (high quality) or the low end (low quality). Resulting profits are given by the following payoff matrix:

		Firm 2	
		Low	***High***
Firm 1	***Low***	**-20, -30**	**900, 600**
	High	**100, 800**	**50, 50**

a. What outcomes, if any, are Nash equilibria?

A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. If Firm 2 chooses Low and Firm 1 chooses High, neither will have an incentive to change (100 > -20 for Firm 1 and 800 > 50 for Firm 2). If Firm 2 chooses High and Firm 1 chooses Low, neither will have an incentive to change (900 > 50 for Firm 1 and 600 > -30 for Firm 2). Both outcomes are Nash equilibria. Both firms choosing low is not a Nash equilibrium because, for example, if Firm 1 chooses low then firm 2 is better off by switching to high since 600 is greater than -30.

b. If the manager of each firm is conservative and each follows a maximin strategy, what will be the outcome?

If Firm 1 chooses Low, its worst payoff, -20, would occur if Firm 2 chooses Low. If Firm 1 chooses High, its worst payoff, 50, would occur if Firm 2 chooses High. Therefore, with a conservative maximin strategy, Firm 1 chooses High. Similarly, if Firm 2 chooses Low, its worst payoff, -30, would occur if Firm 1 chooses Low. If Firm 2 chooses High, its worst payoff, 50, would occur if Firm 1 chooses High. Therefore, with a maximin strategy, Firm 2 chooses High. Thus, both firms choose High, yielding a payoff of 50 for both.

c. What is the cooperative outcome?

The cooperative outcome would maximize *joint* payoffs. This would occur if Firm 1 goes for the low end of the market and Firm 2 goes for the high end of the market. The joint payoff is 1,500 (Firm 1 gets 900 and Firm 2 gets 600).

d. Which firm benefits most from the cooperative outcome? How much would that firm need to offer the other to persuade it to collude?

Firm 1 benefits most from cooperation. The difference between its best payoff under cooperation and the next best payoff is 900 - 100 = 800. To persuade Firm 2 to choose Firm 1's best option, Firm 1 must offer at least the difference between Firm 2's payoff under cooperation, 600, and its best payoff, 800, i.e., 200. However, Firm 2 realizes that Firm 1 benefits much more from cooperation and should try to extract as much as it can from Firm 1 (up to 800).

5. Two major networks are competing for viewer ratings in the 8:00-9:00 P.M. and 9:00-10:00 P.M. slots on a given weeknight. Each has two shows to fill this time period and is juggling its lineup. Each can choose to put its "bigger" show first or to place it second in the 9:00-10:00 P.M. slot. The combination of decisions leads to the following "ratings points" results:

		Network 2	
		First	**Second**
Network 1	**First**	**15, 15**	**30, 10**
	Second	**20, 30**	**18, 18**

a. Find the Nash equilibria for this game, assuming that both networks make their decisions at the same time.

A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. By inspecting each of the four combinations, we find that (Second, First) is the only Nash equilibrium, yielding a payoff of (20, 30). There is no incentive for either party to change from this outcome. If we pick First for Firm 1 and Second for Firm 2, Firm 2 has an incentive to switch to First, in which case Firm 1 is better switching to Second.

b. If each network is risk averse and uses a maximin strategy, what will be the resulting equilibrium?

This conservative strategy of minimizing the maximum loss focuses on limiting the extent of the worst possible outcome, to the exclusion of possible good outcomes. If Network 1 plays First, the worst payoff is 15. If Network 1 plays Second, the worst payoff is 18. Under maximin, Network 1 plays Second. If Network 2 plays First, the worst payoff is 15. If Network 2 plays Second, the worst payoff is 10. Under maximin, Network 2 plays First (This is a dominant strategy). The maximin equilibrium is (Second, First) with a payoff of (20,30).

c. What will be the equilibrium if Network 1 can makes its selection first? If Network 2 goes first?

If Network 1 plays First, Network 2 will play First, yielding 15 for Network 1. If Network 1 plays Second, Network 2 will play First, yielding 20 for Network 1. Therefore, if it has the first move, Network 1 will play Second, and the resulting equilibrium will be (Second, First). If Network 2 plays First, Network 1 will play Second, yielding 30 for Network 2. If Network 2 plays Second, Network 1 will play First, yielding 10 for Network 2. If it has the first move, Network 2 will play First, and the equilibrium will again be (Second, First).

d. Suppose the network managers meet to coordinate schedules and Network 1 promises to schedule its big show first. Is this promise credible? What would be the likely outcome?

A move is credible if, once declared, *there is no incentive to change*. If Network 1 goes first, then Network 2 will also want to go first which gives them both 15. In this case, once Network 1 knows that Network 2 also wants to go first, Network 1 will wants to change its strategy to Second. In this case, the promise to schedule the bigger show first is not credible. Network 2 will schedule its bigger show First since this is a dominant strategy and the coordinated outcome is likely to be (Second,First).

6. Two competing firms are each planning to introduce a new product. Each will decide whether to produce Product *A*, Product *B*, or Product *C*. They will make their choices at the same time. The resulting payoffs are shown below.

We are given the following payoff matrix, which describes a product introduction game:

		Firm 2		
		A	*B*	*C*
	A	-10,-10	0,10	10,20
Firm 1	*B*	10,0	-20,-20	-5,15
	C	20,10	15,-5	-30,-30

a. Are there any Nash equilibria in pure strategies? If so, what are they?

There are two Nash equilibria in pure strategies. Each one involves one firm introducing Product A and the other firm introducing Product C. We can write these two strategy pairs as (A, C) and (C, A), where the first strategy is for player 1. The payoff for these two strategies is, respectively, (10,20) and (20,10).

b. If both firms use *maximin* strategies, what outcome will result?

Recall that maximin strategies maximize the minimum payoff for both players. For each of the players the strategy that maximizes their minimum payoff is A. Thus (A,A) will result, and payoffs will be (-10,-10). Each player is much worse off than at either of the pure strategy Nash equilibrium.

c. If Firm 1 uses a maximin strategy and Firm 2 knows, what will Firm 2 do?

If Firm 1 plays its maximin strategy of A, and Firm 2 knows this then Firm 2 would get the highest payoff by playing C. Notice that when Firm 1 plays conservatively, the Nash equilibrium that results gives Firm 2 the highest payoff of the two Nash equilibria.

7. We can think of the U.S. and Japanese trade policies as a prisoners' dilemma. The two countries are considering policies to open or close their import markets. Suppose the payoff matrix is:

		Japan	
		Open	**Close**
	Open	**10, 10**	**5, 5**
U.S.	**Close**	**-100, 5**	**1, 1**

a. Assume that each country knows the payoff matrix and believes that the other country will act in its own interest. Does either country have a dominant strategy? What will be the equilibrium policies if each country acts rationally to maximize its welfare?

Choosing Open is a dominant strategy for both countries. If Japan chooses Open, the U.S. does best by choosing Open. If Japan chooses Close, the U.S. does best by choosing Open. Therefore, the U.S. should choose Open, no matter what Japan does. If the U.S. chooses Open, Japan does best by choosing Open. If the U.S. chooses Close, Japan does best by choosing Open. Therefore, both countries will choose to have Open policies in equilibrium.

b. Now assume that Japan is not certain that the U.S. will behave rationally. In particular, Japan is concerned that U.S. politicians may want to penalize Japan even if that does not maximize U.S. welfare. How might this affect Japan's choice of strategy? How might this change the equilibrium?

The irrationality of U.S. politicians could change the equilibrium to (Close, Open). If the U.S. wants to penalize Japan they will choose Close, but Japan's strategy will not be affected since choosing Open is still Japan's dominant strategy.

8. You are a duopolist producer of a homogeneous good. Both you and your competitor have *zero* marginal costs. The market demand curve is

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Q_1 is your output and Q_2 your competitor's output. Your competitor has also read this book.

a. Suppose you will play this game only once. If you and your competitor must announce your output at the same time, how much will you choose to produce? What do you expect your profit to be? Explain.

These are some of the cells in the payoff matrix:

Firm 2's Output

Firm 1's Output	0	5	10	15	20	25	30
0	0,0	0,125	0,200	0,225	0,200	0,125	0,0
5	125,0	100,100	75,150	50,150	25,100	0,0	0,0
10	200,0	150,75	100,100	50,75	0,0	0,0	0,0
15	225,0	100,50	75,50	0,0	0,0	0,0	0,0
20	200,0	100,25	0,0	0,0	0,0	0,0	0,0
25	125,0	0,0	0,0	0,0	0,0	0,0	0,0
30	0,0	0,0	0,0	0,0	0,0	0,0	0,0

If both firms must announce output at the same time, both firms believe that the other firm is behaving rationally, and each firm treats the output of the other firm as a fixed number, a Cournot equilibrium will result.

For Firm 1, total revenue will be

$$TR_1 = (30 - (Q_1 + Q_2))Q_1, \quad \text{or} \quad TR_1 = 30Q_1 - Q_1^2 - Q_1Q_2.$$

Marginal revenue for Firm 1 will be the derivative of total revenue with respect to Q_1,

$$\frac{\partial TR}{\partial Q_1} = 30 - 2Q_1 - Q_2.$$

Because the firms share identical demand curves, the solution for Firm 2 will be symmetric to that of Firm 1:

$$\frac{\partial TR}{\partial Q_2} = 30 - 2Q_2 - Q_1.$$

To find the profit-maximizing level of output for both firms, set marginal revenue equal to marginal cost, which is zero:

$$Q_1 = 15 - \frac{Q_2}{2} \quad \text{and}$$

$$Q_2 = 15 - \frac{Q_1}{2}.$$

With two equations and two unknowns, we may solve for Q_1 and Q_2:

$$Q_1 = 15 - (0.5)\left(15 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 10.$$

By symmetry, $Q_2 = 10$.

Substitute Q_1 and Q_2 into the demand equation to determine price:

$$P = 30 - (10 + 10), \text{ or } P = \$10.$$

Since no costs are given, profits for each firm will be equal to total revenue:

$$\pi_1 = TR_1 = (10)(10) = \$100 \quad \text{and}$$
$$\pi_2 = TR_2 = (10)(10) = \$100.$$

Thus, the equilibrium occurs when both firms produce 10 units of output and both firms earn \$100. Looking back at the payoff matrix, note that the outcome (100, 100) is indeed a Nash equilibrium: neither firm will have an incentive to deviate, given the other firm's choice.

b. Suppose you are told that you must announce your output *before* your competitor does. How much will you produce in this case, and how much do you think your competitor will produce? What do you expect your profit to be? Is announcing first an advantage or disadvantage? Explain briefly. *How much would you pay* to be given the option of announcing either first or second?

If you must announce first, you would announce an output of 15, knowing that your competitor would announce an output of 7.5. (Note: This is the Stackelberg equilibrium.)

$$TR_1 = (30 - (Q_1 + Q_2))Q_1 = 30Q_1 - Q_1^2 - Q_1\left(15 - \frac{Q_1}{2}\right) = 15Q_1 - \frac{Q_1^2}{2}.$$

Therefore, setting $MR = MC = 0$ implies:

$$15 - Q_1 = 0, \text{ or } Q_1 = 15 \quad \text{and}$$
$$Q_2 = 7.5.$$

At that output, your competitor is maximizing profits, given that you are producing 15. At these outputs, price is equal to

$$30 - 15 - 7.5 = \$7.5.$$

Your profit would be

$$(15)(7.5) = \$112.5.$$

Your competitor's profit would be

$$(7.5)(7.5) = \$56.25.$$

Announcing first is an advantage in this game. The difference in profits between announcing first and announcing second is \$56.25. You would be willing to pay up to this difference for the option of announcing first.

c. Suppose instead that you are to play the first round of a *series of 10 rounds* (with the same competitor). In each round, you and your competitor announce your outputs at the same time. You want to maximize the sum of your profits over the 10 rounds. How much will you produce in the *first round*? How much do you expect to produce in the tenth round? In the ninth round? Explain briefly.

Given that your competitor has also read this book, you can assume that he or she will be acting rationally. You should begin with the Cournot output and continue with the

Cournot output in each round, including the ninth and tenth rounds. Any deviation from this output will reduce the sum of your profits over the ten rounds.

d. Once again you will play a series of 10 rounds. This time, however, in each round your competitor will announce its output before you announce yours. How will your answers to (c) change in this case?

If your competitor always announces first, it might be more profitable to behave by reacting "irrationally" in a single period. For example, in the first round your competitor will announce an output of 15, as in Exercise (7.b). Rationally, you would respond with an output of 7.5. If you behave this way in every round, your total profits for all ten rounds will be $562.50. Your competitor's profits will be $1,125. However, if you respond with an output of 15 every time your competitor announces an output of 15, profits will be reduced to zero for both of you in that period. If your competitor fears, or learns, that you will respond in this way, he or she will be better off by choosing the Cournot output of 10, and your profits after that point will be $75 per period. Whether this strategy is profitable depends on your opponent's expectations about your behavior, as well as how you value future profits relative to current profits.

(Note: A problem could develop in the last period, however, because your competitor will know that you realize that there are no more long-term gains to be had from behaving strategically. Thus, your competitor will announce an output of 15, knowing that you will respond with an output of 7.5. Furthermore, knowing that you will not respond strategically in the last period, there are also no long-term gains to be made in the ninth period from behaving strategically. Therefore, in the ninth period, your competitor will announce an output of 15, and you should respond rationally with an output of 7.5, and so on.)

9. You play the following bargaining game. Player A moves first and makes Player B an offer for the division of $100. (For example, Player A could suggest that she take $60 and Player B take $40). Player B can accept or reject the offer. If he rejects it, the amount of money available drops to $90, and he then makes an offer for the division of this amount. If Player A rejects this offer, the amount of money drops to $80 and Player A makes an offer for its division. If Player B rejects this offer, the amount of money drops to 0. Both players are rational, fully informed, and want to maximize their payoffs. Which player will do best in this game?

Solve the game by starting at the end and working backwards. If B rejects A's offer at the 3rd round, B gets 0. When A makes an offer at the 3rd round, B will accept even a minimal amount, such as $1. So A should offer $1 at this stage and take $79 for herself. In the 2nd stage, B knows that A will turn down any offer giving her less than $79, so B must offer $80 to A, leaving $10 for B. At the first stage, A knows B will turn down any offer giving him less than $10. So A can offer $11 to B and keep $89 for herself. B will take that offer, since B can never do any better by rejecting and waiting. The following table summarizes this.

Round	Money Available	Offering Party	Amount to A	Amount to B
1	$100	A	$89	$11
2	$ 90	B	$80	$10
3	$ 80	A	$79	$ 1
End	$ 0		$ 0	$ 0

***10. Defendo has decided to introduce a revolutionary video game. As the first firm in the market, it will have a monopoly position for at least some time. In deciding what type of manufacturing plant to build, it has the choice of two technologies. Technology A is publicly available and will result in annual costs of**

$$C^A(q) = 10 + 8q.$$

Technology B is a proprietary technology developed in Defendo's research labs. It involves higher fixed cost of production but lower marginal costs:

$$C^B(q) = 60 + 2q.$$

Defendo must decide which technology to adopt. Market demand for the new product is P = 20 - Q, where Q is total industry output.

a. Suppose Defendo were certain that it would maintain its monopoly position in the market for the entire product lifespan (about five years) without threat of entry. Which technology would you advise Defendo to adopt? What would be Defendo's profit given this choice?

Defendo has two choices: Technology A with a marginal cost of 8 and Technology B with a marginal cost of 2. Given the inverse demand curve as $P = 20 - Q$, total revenue, PQ, is equal to $20Q - Q^2$ for both technologies. Marginal revenue is $20 - 2Q$. To determine the profits for each technology, equate marginal revenue and marginal cost:

$$20 - 2Q_A = 8, \text{ or } Q_A = 6, \quad \text{and}$$
$$20 - 2Q_B = 2, \text{ or } Q_B = 9.$$

Substituting the profit-maximizing quantities into the demand equation to determine the profit-maximizing prices, we find:

$$P_A = 20 - 6 = \$14 \quad \text{and}$$
$$P_B = 20 - 9 = \$11.$$

To determine the profits for each technology, subtract total cost from total revenue:

$$\pi_A = (14)(6) - (10 + (8)(6)) = \$26 \quad \text{and}$$
$$\pi_B = (11)(9) - (60 + (2)(9)) = \$21.$$

To maximize profits, Defendo should choose technology A.

b. Suppose Defendo expects its archrival, Offendo, to consider entering the market shortly after Defendo introduces its new product. Offendo will have access only to Technology A. If Offendo does enter the market, the two firms will play a Cournot game (in quantities) and arrive at the Cournot-Nash equilibrium.

i. If Defendo adopts Technology A and Offendo enters the market, what will be the profit of each firm? Would Offendo choose to enter the market given these profits?

If both firms play Cournot, each will choose its best output, taking the other's strategy as given. Letting D = Defendo and O = Offendo, the demand function will be

$$P = 20 - Q_D - Q_O.$$

Profit for Defendo will be

$$\pi_D = (20 - Q_D - Q_O)Q_D - (10 + 8Q_D), \text{ or } \pi_D = 12Q_D - Q_D^2 - Q_D Q_O - 10$$

To determine the profit-maximizing quantity, set the first derivative of profits with respect to Q_D equal to zero and solve for Q_D:

$$\frac{\partial \pi_D}{\partial Q_D} = 12 - 2Q_D - Q_O = 0 \text{, or } Q_D = 6 - 0.5Q_O.$$

This is Defendo's reaction function. Because both firms have access to the same technology, hence the same cost structure, Offendo's reaction function is analogous:

$$Q_O = 6 - 0.5Q_D.$$

Substituting Offendo's reaction function into Defendo's reaction function and solving for Q_D:

$$Q_D = 6 - (0.5)(6 - 0.5Q_D) = 4.$$

Substituting into Defendo's reaction function and solving for Q_O:

$$Q_O = 6 - (0.5)(4) = 4.$$

Total industry output is therefore equal to 8. To determine price, substitute Q_D and Q_O into the demand function:

$$P = 20 - 4 - 4 = \$12.$$

The profits for each firm are equal to total revenue minus total costs:

$$\pi_D = (4)(12) - (10 + (8)(4)) = \$6 \quad \text{and}$$
$$\pi_O = (4)(12) - (10 + (8)(4)) = \$6.$$

Therefore, Offendo would enter the market.

ii. If Defendo adopts Technology B and Offendo enters the market, what will be the profit of each firm? Would Offendo choose to enter the market given these profits?

Profit for Defendo will be

$$\pi_D = (20 - Q_D - Q_O)Q_D - (60 + 2Q_D), \text{ or } \pi_D = 18Q_D - Q_D^2 - Q_D Q_O - 60.$$

The change in profit with respect to Q_D is

$$\frac{\partial \pi_D}{\partial Q_D} = 18 - 2Q_D - Q_O.$$

To determine the profit-maximizing quantity, set this derivative to zero and solve for Q_D:

$$18 - 2Q_D - Q_O = 0, \text{ or } Q_D = 9 - 0.5Q_O.$$

This is Defendo's reaction function. Substituting Offendo's reaction function (from part i above) into Defendo's reaction function and solving for Q_D:

$$Q_D = 9 - 0.5(6 - 0.5Q_D), \text{ or } Q_D = 8.$$

Substituting Q_D into Offendo's reaction function yields

$$Q_O = 6 - (0.5)(8), \text{ or } Q_O = 2.$$

To determine the industry price, substitute the profit-maximizing quantities for Defendo and Offendo into the demand function:

$$P = 20 - 8 - 2 = \$10.$$

The profit for each firm is equal to total revenue minus total cost, or:

$$\pi_D = (10)(8) - (60 + (2)(8)) = \$4 \quad \text{and}$$
$$\pi_O = (10)(2) - (10 + (8)(2)) = -\$6.$$

With negative profit, Offendo *should not* enter the industry.

iii. Which technology would you advise Defendo to adopt given the threat of possible entry? What will be Defendo's profit given this choice? What will be consumer surplus given this choice?

With Technology A and Offendo's entry, Defendo's profit would be 6. With Technology B and no entry by Defendo, Defendo's profit would be 4. I would advise Defendo to stick with Technology A. Under this advice, total output is 8 and price is 12. Consumer surplus is:

$$(0.5)(20 - 12)(8) = \$32.$$

c. What happens to social welfare (the sum of consumer surplus and producer profit) as a result of the threat of entry in this market? What happens to equilibrium price? What might this imply about the role of *potential* competition in limiting market power?

From 10.a we know that, under monopoly, $Q = 6$ and profit is 26. Consumer surplus is

$$(0.5)(20 - 14)(6) = \$18.$$

Social welfare is the sum of consumer surplus plus profits, or

$$18 + 26 = \$44.$$

With entry, social welfare is \$32 (consumer surplus) plus \$12 (industry profit), or \$44. Social welfare does not change with entry, but entry shifts surplus from producers to consumers. The equilibrium price falls with entry, and therefore *potential* competition can *limit* market power.

Note that Defendo has one other option: to increase quantity from the monopoly level of 6 to discourage entry by Offendo. If Defendo increases output from 6 to 8 under Technology A, Offendo is unable to earn a positive profit. With an output of 8, Defendo's profit decreases from \$26 to

$$(8)(12) - (10 + (8)(8)) = \$22.$$

As before, with an output of 8, consumer surplus is \$32; social welfare is \$54. In this case, social welfare rises when output is increased to discourage entry.

11. Three contestants, A, B, and C, each have a balloon and a pistol. From fixed positions, they fire at each other's balloons. When a balloon is hit, its owner is out. When only one balloon remains, its owner is the winner of a $1,000 prize. At the outset, the players decide by lot the order in which they will fire, and each player can choose any remaining balloon as his target. Everyone knows that A is the best shot and always hits the target, that B hits the target with probability .9, and that C hits the target with probability 0.8. Which contestant has the highest probability of winning the $1,000? Explain why.

Intuitively, C has the highest probability of winning, though A has the highest probability of shooting the balloon. Each contestant wants to remove the contestant with the highest probability of success. By following this strategy, each improves his chance of winning the game. A targets B because, by removing B from the game, A's chance of winning becomes much greater. B's probability of success is greater than C's probability of success. C will target A because, if C targets B and hits B, then A will target C and win the game. B will follow a similar strategy, because if B targets C and hits C, then A will target B and will win the game. Therefore, both B and C increase their chance of winning by eliminating A first. Similarly, A increases his chance of winning by eliminating B first. A complete probability tree can be constructed to show that A's chance of winning is 8 percent, B's chance of winning is 32 percent, and C's chance of winning is 60 percent.

12. An antique dealer regularly buys objects at home-town auctions whose bidders are limited to other dealers. Most of her successful bids turn out to be financially worthwhile, because she is able to resell the antiques for a profit. On occasion, however, she travels to a nearby town to bid in an auction that is open to the public. She often finds that on the rare occasions in which she does bid successfully, she is disappointed - the antique cannot be sold at a profit. Can you explain the difference in her success between the two sets of circumstances?

When she bids at the home-town auction that is limited to other dealers, she is bidding against people who are all going to resell the antique if they win the bid. In this case, all of the bidders are limiting their bids to prices that will tend to earn them a profit. A rational dealer will not place a bid which is higher than the price they can expect to resell the antique for. Given that all dealers are rational, the winning bid will tend to be below the expected resale price.

When she bids in the auction that is open to the public she is bidding against the people who are likely to come into her shop. You can assume that local antique lovers will frequent these auctions as well as the local antique shops. In the case where she wins the bid at one of these open auctions, the other participants have decided that the price is too high. In this case, they will not come into her shop and pay any higher price which would earn her a profit. She will only tend to profit in this case if she is able to resell to a customer from out of the area, or who was not at the auction, and who has a sufficiently high reservation price. In any event, the winning bid price will tend to be higher because she was bidding against customers rather than dealers.

13. You are in the market for a new house and have decided to bid for a house at auction. You believe that the value of the house is between $125,000 and $150,000, but you are uncertain as to where in the range it might be. You do know, however, that the seller has reserved the right to withdraw the house from the market if the winning bid is not satisfactory.

a. Should you bid in this auction? Why or why not?

Yes you should bid if you are confident about your estimate of the value of the house and/or if you allow for the possibility of being wrong. To allow for the possibility of being wrong, you reduce your high bid by an amount equal to the expected error of the winning bidder. If you have experience at auction, you will have information on how likely you are to enter a wrong bid and can then adjust your high bid accordingly.

b. Suppose you are a building contractor. You plan to improve the house and then to resell it at a profit. How does this situation affect your answer to (a)? Does it depend on the extent to which your skills are uniquely suitable to improving this particular house?

You need to be aware of the winner's curse which says that the winner is likely to be the person who most overestimated the value of the house. If there are a range of bids, some below the actual value and some above the actual value, the winner will be the person with the largest overestimate of the value. Again, you need to be very confident in your estimate of the value of the house and/or allow for the possibility of being wrong. If you have made many such bids in the past, you will be able to estimate how often you are wrong and adjust accordingly.