1. A firm uses cloth and labor to produce shirts in a factory that it bought for $10 million. Which of its factor inputs are measured as flows and which as stocks? How would your answer change if the firm had leased a factory instead of buying one? Is its output measured as a flow or a stock? What about profit?

Inputs that are purchased or used up during a particular time period are flows. Flow variables can be measured in terms of hours, days, weeks, months, or years. Inputs measured at a particular point in time are stocks. All stock variables have an associated flow variable. At any particular time, a firm will have a stock of buildings and machines that it owns. This is the stock variable. During some given time period, the firm may elect to buy a new piece of equipment (this is a flow) or it may depreciate its existing capital resources (this is a flow). In this example, cloth and labor are flows, while the factory is a stock. If the firm instead leases the building, then the factory is still a stock variable that is owned in this case by someone other than the firm. The firm would pay rent during a particular time period, which would be a flow. Output is always a flow variable that is measured over some given time period. Since profit is the difference between the revenues and costs over some given time period, it is also a flow.

2. How do investors calculate the present value of a bond? If the interest rate is 5 percent, what is the present value of a perpetuity that pays $1,000 per year forever?

The present value of a bond is the sum of discounted values of each payment to the bond holder over the life of the bond. This involves the payment of interest in each period and then the repayment of the principal at the end of the bond's life. A perpetuity involves paying the interest in every future period and no repayment of the principal. The present discounted value of a perpetuity is \[ PDV = \frac{A}{R} \], where \( A \) is the annual payment and \( R \) is the annual interest rate. If \( A = 1,000 \) and \( R = 0.05 \), \[ PDV = \frac{1,000}{0.05} = 20,000 \].

3. What is the effective yield on a bond? How does one calculate it? Why do some corporate bonds have higher effective yields than others?

The effective yield is the interest rate that equates the present value of a bond's payment stream with the bond's market price. The present discounted value of a payment made in the future is

\[ PDV = FV(1 + R)^t, \]

where \( t \) is the length of time before payment. The bond's selling price is its \( PDV \). The payments it makes are the future values, \( FV \), paid in time \( t \). Thus, we must solve for \( R \), which is the bond's effective yield. The effective yield is determined by the interaction of buyers and sellers in the bond market. Some corporate bonds have higher effective yields because they are thought to be a more risky investment, and hence buyers must be rewarded with a higher rate of return so that they will be willing to hold the bonds. Higher rates of return imply a lower present discounted value. If bonds have the same coupon payments, the bonds of the riskiest firms will sell for less than the bonds of the less risky firms.

4. What is the Net Present Value (NPV) criterion for investment decisions? How does one calculate the NPV of an investment project? If all cash flows for a project are certain, what discount rate should be used to calculate NPV?
The Net Present Value criterion for investment decisions is “invest if the present value of the expected future cash flows from the investment is larger than the cost of the investment” (Section 15.4). We calculate the NPV by (1) determining the present discounted value of all future cash flows and (2) subtracting the discounted value of all costs, present and future. To discount both income and cost, the firm should use a discount rate that reflects its opportunity cost of capital, the next highest return on an alternative investment of similar riskiness. Therefore, the risk-free interest rate should be used if the cash flows are certain.

5. You are retiring from your job and are given two options. You can accept a lump sum payment from the company, or you can accept a smaller annual payment that will continue for as long as you live. How would you decide which option is best? What information do you need?

The best option is the one that has in the highest present discounted value. The lump sum payment has a present discounted value equal to the amount of the lump sum payment. To calculate the present discounted value of the payment stream you need to know approximately how many years you might live. If you made a guess of 25 years you could then discount each of the 25 payments back to the current year and add them up to see how this sum compares to the lump sum payment. The discount factor would be the average expected interest rate. Alternatively, you could take the average expected interest rate and compute the annual interest that could be earned from the lump sum and see how this interest amount compares to the annual payment. For example, if the sum is $600,000 and the interest rate is 8% then the annual interest is $48,000. This means you could live off of the $48,000 and never touch the principal. The annual payment would need to be greater than $48,000 in this case to make it worthwhile. Finally, you must consider the time and risk involved in managing a lump sum and decide if it is better or easier to just take the smaller annual payment.

6. You have noticed that bond prices have been rising over the past few months. All else equal, what does this suggest has been happening to interest rates? Explain.

This suggests that interest rates have been falling because bond prices and interest rates are inversely related. When the price of a bond (with a fixed coupon payment) rises, then the effective yield on the bond will fall. The only way people will be willing to hold the bond is if interest rates in general are also falling. If interest rates are lower than the effective yield on the bond for example, then people will prefer to hold the bond. When more people move into bonds, the price of the bond will rise and the effective yield will fall. Bond prices therefore adjust to bring the effective yield in line with interest rates.

7. What is the difference between a real discount rate and a nominal discount rate? When should a real discount rate be used in an NPV calculation and when should a nominal rate be used?

The real discount rate is net of inflation, whereas the nominal discount rate includes inflationary expectations. The real discount rate is equal to the nominal discount rate minus the rate of inflation. If cash flows are in real terms, the appropriate discount rate is the real rate. For example, in applying the NPV criterion to a manufacturing decision, if future prices of inputs and outputs are not adjusted for inflation (which they often are not), a nominal discount rate should be used to determine whether the NPV is positive. In sum, all numbers should either be expressed in real terms or nominal terms, but not a mix.

8. How is a risk premium used to account for risk in NPV calculations? What is the difference between diversifiable and nondiversifiable risk? Why should only nondiversifiable risk enter into the risk premium?

To determine the present discounted value of a cash flow, the discount rate should reflect the riskiness of the project generating the cash flow. The risk premium is the
difference between a discount rate that reflects the riskiness of the cash flow and a discount rate on a risk-free flow, e.g., the discount rate associated with a short-term government bond. The higher the riskiness of a project, the higher the risk premium.

Diversifiable risk can be eliminated by investing in many projects. Hence, an efficient capital market will not compensate an investor for taking on risk that can be eliminated costlessly. Nondiversifiable risk is that part of a project’s risk that cannot be eliminated by investing in a large number of other projects. It is that part of a project’s risk which is correlated with the portfolio of all projects available in the market. Since investors can eliminate diversifiable risk, they cannot expect to earn a risk premium on diversifiable risk.

9. What is meant by the “market return” in the Capital Asset Pricing Model (CAPM)? Why is the market return greater than the risk-free interest rate? What does an asset’s “beta” measure in the CAPM? Why should high-beta assets have a higher expected return than low-beta assets?

In the Capital Asset Pricing Model (CAPM), the market return is the rate of return on the portfolio of assets held by the market. The market return reflects nondiversifiable risk.

Since the market portfolio has no diversifiable risk, the market return reflects the risk premium associated with holding one unit of nondiversifiable risk. The market rate of return is greater than the risk-free rate of return, because risk-averse investors must be compensated with higher average returns for holding a risky asset.

An asset’s beta reflects the sensitivity (covariance) of the asset’s return with the return on the market portfolio. An asset with a high beta will have a greater expected return than a low-beta asset, since the high-beta asset has greater nondiversifiable risk than the low-beta asset.

10. Suppose you are deciding whether to invest $100 million in a steel mill. You know the expected cash flows for the project, but they are risky — steel prices could rise or fall in the future. How would the CAPM help you select a discount rate for an NPV calculation?

To evaluate the net present value of a $100 million investment in a steel mill, you should use the stock market’s current evaluation of firms that own steel mills as a guide to selecting the appropriate discount rate. For example, you would (1) identify nondiversified steel firms, those that are primarily involved in steel production, (2) determine the beta associated with stocks issued by those companies (this can be done statistically or by relying on a financial service that publishes stock betas, such as Value Line), and (3) take a weighted average of these betas, where the weights are equal to the firm’s assets divided by the sum of all diversified steel firms’ assets. With an estimate of beta, plus estimates of the expected market and risk-free rates of return, you could infer the discount rate using Equation (15.7) in the text: Discount rate = \( r_f + \beta (r_m + r_f) \).

11. How does a consumer trade off current and future costs when selecting an air conditioner or other major appliance? How could this selection be aided by an NPV calculation?

The NPV calculation for a durable good involves discounting to the present all future services from the appliance, as well as any salvage value at the end of the appliance’s life, and subtracting its cost and the discounted value of any expenses. Discounting is done at the opportunity cost of money. Of course, this calculation assumes well-defined quantities of future services. If these services are not well defined, the consumer should ask what value of these services would yield an NPV of zero. If this value is less than the price that the consumer would be willing to pay in each period, the investment should be made.
12. What is meant by the “user cost” of producing an exhaustible resource? Why does price minus extraction cost rise at the rate of interest in a competitive exhaustible resource market?

In addition to the opportunity cost of extracting the resource and preparing it for sale, there is an additional opportunity cost arising from the depletion of the resource. User cost is the difference between price and the marginal cost of production. User cost rises over time because as reserves of the resource become depleted, the remaining reserves become more valuable.

Given constant demand over time, the price of the resource minus its marginal cost of extraction, \( P - MC \), should rise over time at the rate of interest. If \( P - MC \) rises faster than the rate of interest, no extraction should occur in the present period, because holding the resource for another year would earn a higher rate of return than selling the resource now and investing the proceeds for another year. If \( P - MC \) rises slower than the rate of interest, current extraction should increase, thus increasing the supply at each price, lowering the equilibrium price, and decreasing the return on producing the resource. In equilibrium, the price of a resource rises at the rate of interest.

13. What determines the supply of loanable funds? The demand for loanable funds? What might cause the supply or demand for loanable funds to shift? How would such a shift affect interest rates?

The supply of loanable funds is determined by the interest rate offered to savers. A higher interest rate induces households to consume less today (save) in favor of greater consumption in the future. The demand for loanable funds comes from consumers who wish to consume more today than tomorrow or from investors who wish to borrow money. Demand depends on the interest rate at which these two groups can borrow. Several factors can shift the demand and supply of loanable funds. On the one hand, for example, a recession decreases demand at all interest rates, shifting the demand curve inward and causing the equilibrium interest rate to fall. On the other hand, the supply of loanable funds will shift out if the Federal Reserve increases the money supply, again causing the interest rate to fall.

EXERCISES

1. Suppose the interest rate is 10 percent. If $100 is invested at this rate today, how much will it be worth after one year? After two years? After five years? What is the value today of $100 paid one year from now? Paid two years from now? Paid five years from now?

We would like to know the future value, \( FV \), of $100 invested today at an interest rate of 10 percent. One year from now our investment will be equal to

\[
FV = $100 + ($100)(10\%) = $110.
\]

Two years from now we will earn interest on the $100 ($10) and we will earn interest on the interest from the first year, i.e., ($10)(10\%) = $1. Thus, our investment will be worth $100 + $10 (from the first year) + $10 (from the second year) + $1 (interest on the first year’s interest) = $121.

Algebraically, \( FV = PDV(1 + R)^t \), where \( PDV \) is the present discounted value of the investment, \( R \) is the interest rate, and \( t \) is the number of years. After two years,

\[
FV = PDV(1 + R)^t = ($100)(1.1)^2 = ($100)(1.21) = $121.00.
\]

After five years

\[
FV = PDV(1 + R)^t = ($100)(1.1)^5 = ($100)(1.61051) = $161.05.
\]
To find the present discounted value of $100 paid one year from now, we ask how much is needed to invest today at 10 percent to have $100 one year from now. Using our formula, we solve for PDV as a function of FV:

$$PDV = (FV)(1 + R)^t.$$  

With $t = 1$, $R = 0.10$, and $FV = $100,

$$PDV = (100)(1.1)^1 = $90.91.$$  

With $t = 2$, $PDV = (1.1)^2 = $82.64,

With $t = 5$, $PDV = (1.1)^5 = $62.09.$$

2. You are offered the choice of two payment streams: (a) $150 paid one year from now and $150 paid two years from now; (b) $130 paid one year from now and $160 paid two years from now. Which payment stream would you prefer if the interest rate is 5 percent? If it is 15 percent?  

To compare two income streams, we calculate the present discounted value of each and choose the stream with the highest present discounted value. We use the formula $PDV = FV(1 + R)^t$ for each cash flow. See Exercise (2) above. Stream (a) has two payments:

$$PDV_a = FV_1(1 + R)^1 + FV_2(1 + R)^2$$

$$PDV_a = ($150)(1.05)^1 + ($150)(1.05)^2,$$ or

$$PDV_a = $142.86 + 136.05 = $278.91.$$  

Stream (b) has two payments:

$$PDV_b = ($130)(1.05)^1 + ($160)(1.05)^2,$$ or

$$PDV_b = $123.81 + $145.12 = $268.93.$$  

At an interest rate of 5 percent, you should select (b).  

If the interest rate is 15 percent, the present discounted values of the two income streams would be:

$$PDV_a = ($150)(1.15)^1 + ($150)(1.15)^2,$$ or

$$PDV_a = $130.43 + $113.42 = $243.85,$$ and

$$PDV_b = ($130)(1.15)^1 + ($160)(1.15)^2,$$ or

$$PDV_b = $113.04 + $120.98 = $234.02.$$  

You should still select (b).

3. Suppose the interest rate is 10 percent. What is the value of a coupon bond that pays $80 per year for each of the next five years and then makes a principal repayment of $1,000 in the sixth year? Repeat for an interest rate of 15 percent.  

We need to determine the present discounted value, PDV, of a stream of payments over the next six years. We translate future values, FV, into the present with the following formula:

$$PDV = \frac{FV}{(1 + R)^t},$$

where $R$ is the interest rate, equal to 10 percent, and $t$ is the number of years in the future. For example, the present value of the first $80 payment one year from now is

$$PDV = \frac{80}{(1 + 0.10)^1} = $72.73.$$  

...
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\[ PDV = \frac{FV}{(1 + R)^t} = \frac{80}{(1 + 0.10)^t} = \frac{80}{1.1} = 72.73. \]

The value of all coupon payments over five years can be found the same way:

\[ PDV = 80 \frac{1}{1.1} + 80 \frac{1}{1.1^2} + 80 \frac{1}{1.1^3} + 80 \frac{1}{1.1^4} + 80 \frac{1}{1.1^5}, \text{ or} \]

\[ PDV = 80 \left( \frac{1}{1.1} + \frac{1}{1.21} + \frac{1}{1.331} + \frac{1}{1.4641} + \frac{1}{1.61051} \right) = 303.26. \]

Finally, we calculate the present value of the final payment of $1,000 in the sixth year:

\[ PDV = \frac{1,000}{1.1^6} = \frac{1,000}{1.771} = 564.47. \]

Thus, the present value of the bond is $303.26 + $564.47 = $867.73.

With an interest rate of 15 percent, we calculate the value of the bond in the same way:

\[ PDV = 80(0.870 + 0.756 + 0.658 + 0.572 + 0.497) + (1,000)(0.432), \text{ or} \]

\[ PDV = 268.17 + 432.32 = 700.49. \]

As the interest rate increases, while payments are held constant, the value of the bond decreases.

4. A bond has two years to mature. It makes a coupon payment of $100 after one year and both a coupon payment of $100 and a principal repayment of $1,000 after two years. The bond is selling for $966. What is its effective yield?

We want to know the interest rate that will yield a present value of $966 for an income stream of $100 after one year and $1,100 after two years. Find \( i \) such that

\[ 966 = (100)(1 + i)^{-1} + (1,100)(1 + i)^{-2}. \]

Algebraic manipulation yields

\[ 966(1 + i)^2 = 100(1 + i) + 1,100, \text{ or} \]

\[ 966 + 1,932i + 966i^2 - 100 - 100i - 1,100 = 0, \text{ or} \]

\[ 966i^2 + 1,832i - 234 = 0. \]

Using the quadratic formula to solve for \( i \),

\[ i = 0.12 \text{ or } -1.068. \]

Since \(-1.068\) does not make economic sense, the effective yield is 12 percent.

5. Equation (15.5) shows the net present value of an investment in an electric motor factory. Half of the $10 million cost is paid initially and the other half after a year. The factory is expected to lose money during its first two years of operation. If the discount rate is 4 percent, what is the NPV? Is the investment worthwhile?

Redefining terms, Equation 15.5 becomes

\[ NPV = -5 - \frac{5}{(1.04)^4} - \frac{1}{(1.04)^2} - \frac{0.5}{(1.04)^3} + \frac{0.96}{(1.04)^4} + \frac{0.96}{(1.04)^5} + \cdots + \frac{0.96}{(1.04)^{20}} + \frac{1}{(1.04)^{20}}. \]

Calculating the \( NPV \) we find:

\[ NPV = -5 - 0.481 - 0.92 - 0.82 + 0.79 + 0.70 + 0.67 + 0.62 + 0.60 + 0.58 + 0.55 + 0.53 + 0.51 + 0.49 + 0.47 + 0.46 + 0.44 + 0.46 = -0.337734. \]

The investment loses $337,734 and is not worthwhile.
6. The market interest rate is 5 percent and is expected to stay at that level. Consumers can borrow and lend all they want at this rate. Explain your choice in each of the following situations:

a. **Would you prefer a $500 gift today or a $540 gift next year?**
   
   The present value of $500 today is $500. The present value of $540 next year is
   \[ \frac{540.00}{1.05} = 514.29 \].
   
   Therefore, I would prefer the $540 next year.

b. **Would you prefer a $100 gift now or a $500 loan without interest for four years?**
   
   If you take the $500 loan, you can invest it for the four years and then pay back the $500. The future value of the $500 is
   \[ 500 \times (1.05)^4 = 607.75 \].
   
   After you pay back the $500 you will have $107.75 left to keep. The future value of the $100 gift is
   \[ 100 \times (1.05)^4 = 121.55 \].
   
   You should take the $100 gift.

c. **Would you prefer a $350 rebate on an $8,000 car or one year of financing for the full price of the car at 0 percent interest?**
   
   The interest rate is 0 percent, which is 5 percent less than the current market rate. You save $400 = (0.05)(8,000) one year from now. The present value of this $400 is
   \[ \frac{400}{1.05} = 380.95 \].
   
   This is greater than $350. Therefore, choose the financing.

d. **You have just won a million dollar lottery and will receive $50,000 a year for the next 20 years. How much is this worth to you today?**
   
   We must find the net present value of $50,000 a year for the next 20 years:
   \[ NPV = 50,000 + \frac{50,000}{(1.05)} + \frac{50,000}{(1.05)^2} + \ldots + \frac{50,000}{(1.05)^{18}} + \frac{50,000}{(1.05)^{19}} = 624,613.54 \]


e. **You win the “honest million” jackpot. You can have $1 million today or $60,000 per year for eternity (a right that can be passed on to your heirs). Which do you prefer?**
   
   The value of the perpetuity is $1,200,000, which makes it advisable take the $60,000 per year payment.

d. **In the past, adult children had to pay taxes on gifts over $10,000 from their parents, but parents could loan money to their children interest-free. Why did some people call this unfair? To whom were the rules unfair?**
   
   Any gift of $N from parent to child could be made without taxation by loaning the child \( \frac{N(1+r)}{r} \). For example, to avoid taxes on a $50,000 gift, the parent would loan the child $550,000, assuming a 10 percent interest rate. With that money, the child could earn $55,000 in interest after one year and still have $500,000 to pay back to the parent. The present value of $55,000 one year from now is $50,000. People of more moderate incomes would find these rules unfair: they might only be able to afford to give the child $50,000 directly, but it would not be tax free.
7. Ralph is trying to decide whether to go to graduate school. If he spends two years in graduate school, paying $15,000 tuition each year, he will get a job that will pay $60,000 per year for the rest of his working life. If he does not go to school, he will go into the work force immediately. He will then make $30,000 per year for the next three years, $45,000 for the following three years, and $60,000 per year every year after that. If the interest rate is 10 percent, is graduate school a good financial investment?

Consider Ralph’s income over the next six years, assuming all payments occur at the end of the year. (After the sixth year, Ralph’s income will be the same with or without education.) With graduate school, the present value of income for the next six years is $113,631,
\[
\text{NPV} = -15,000 \cdot (1.1)^1 + \frac{60,000}{(1.1)^2} + \frac{60,000}{(1.1)^3} + \frac{60,000}{(1.1)^4} + \frac{60,000}{(1.1)^5} + \frac{60,000}{(1.1)^6} = 131,150.35.
\]

Without graduate school, the present value of income for the next six years is
\[
\text{NPV} = -30,000 \cdot (1.1)^1 + \frac{30,000}{(1.1)^2} + \frac{45,000}{(1.1)^3} + \frac{45,000}{(1.1)^4} + \frac{45,000}{(1.1)^5} + \frac{45,000}{(1.1)^6} = 158,683.95.
\]

The payoff from graduate school is not large enough to justify the foregone income and tuition expense while Ralph is in school; he should therefore not go to school.

8. Suppose your uncle gave you an oil well like the one described in Section 15.8. (Marginal production cost is constant at $10.) The price of oil is currently $20 but is controlled by a cartel that accounts for a large fraction of total production. Should you produce and sell all your oil now or wait to produce? Explain your answer.

If a cartel accounts for a large fraction of total production, today’s price minus marginal cost, \( P - MC \) will rise at a rate less than the rate of interest. This is because the cartel will choose output such that marginal revenue minus marginal cost rises at the rate of interest. Since price exceeds marginal revenue, \( P^t - MC \) will rise at a rate less than the rate of interest. So, to maximize net present value, all oil should be sold today. The profits should be invested at the rate of interest.

*9. You are planning to invest in fine wine. Each case of wine costs $100, and you know from experience that the value of a case of wine held for \( t \) years is \( (100)^{1/2} \). One hundred cases of wine are available for sale, and the interest rate is 10 percent.

a. How many cases should you buy, how long should you wait to sell them, and how much money will you receive at the time of their sale?

Buying a case is a good investment if the net present value is positive. If we buy a case and sell it after \( t \) years, we pay $100 now and receive \( 100(1.1)^{0.5} \) when it is sold. The NPV of this investment is
\[
NPV = -100 + e^{-0.1t}\left(100^{0.5}\right) = -100 + e^{-0.1t}\left(100^{0.5}\right).
\]

If we do buy a case, we will choose \( t \) to maximize the NPV. This implies differentiating with respect to \( t \) to obtain the necessary condition that
\[
\frac{dNPV}{dt} = (-0.1)e^{-0.1t}\left(50t^{-0.5}\right) - \left(0.1e^{-0.1t}\right)\left(100^{0.5}\right) = 0.
\]

By multiplying both sides of the first order condition by \( e^{0.1t} \), we obtain
\[
50t^{-0.5} - 10t^{0.5} = 0, \text{ or } t = 5.
\]

If we held the case for 5 years, the NPV is
\[
NPV = -100 + e^{-0.1\times5}\left(100^{0.5}\right) = 35.67.
\]
Therefore, we should buy a case and hold it for five years, where the value at the time of sale is $(100)(5^{0.5})$. Since each case is a good investment, we should buy all 100 cases.

Another way to get the same answer is to compare holding the wine to putting your $100 in the bank. The bank pays interest of 10 percent, while the wine increases in value at the rate of

\[
d\text{value}/dt = 50t^{-0.5} - 0.5\times 100t^{0.5} = \frac{1}{2t}.
\]

As long as \( t < 5 \), the return on wine is greater than or equal to 10 percent. After \( t = 5 \), the return on wine drops below 10 percent. Therefore, \( t = 5 \) is the time to switch your wealth from wine to the bank. As for the issue of whether to buy wine at all, if we put $100 in the bank, we will have \( 100e^{0.5} \) after five years, whereas if we spend $100 on wine, we will have \( 100t^{0.5} = (100)(5^{0.5}) \), which is greater than \( 100e^{0.5} \) in five years.

b. **Suppose that at the time of purchase, someone offers you $130 per case immediately. Should you take the offer?**

You just bought the wine and are offered $130 for resale. You should accept the offer if the \( NPV \) is positive. You get $130 now, but lose the \( (100)(5^{0.5}) \) you would get for selling in five years. Thus, the \( NPV \) of the offer is

\[
NPV = 130 - (e^{0.10(5)})(100)(5^{0.5}) = -238 < 0.
\]

Therefore, you should not sell.

The other approach to solving this problem is to note that the $130 could be put in the bank and would grow to

\[
$214.33 = (130)e^{0.5},
\]

in five years. This is still less than

\[
$223.61 = (100)(5^{0.5}),
\]

the value of the wine after five years.

c. **How would your answers change if the interest rate were only 5 percent?**

If the interest rate changes from 10 percent to 5 percent, the \( NPV \) calculation is

\[
NPV = -100 + (e^{-0.05t})(100)(t^{0.5}).
\]

As before, we maximize this expression:

\[
\frac{dNPV}{dt} = (e^{-0.05t})(50t^{-0.5}) - (0.05)(e^{-0.05t})(100t^{0.5}) = 0.
\]

By multiplying both sides of the first order condition by \( e^{0.05t} \), it becomes

\[
50t^{-0.5} - 5t^{0.5} = 0,
\]

or \( t = 10 \). If we hold the case 10 years, \( NPV \) is

\[
-100 + (e^{-0.05(10)})(100)(10^{0.5}) = $91.80.
\]

With a lower interest rate, it pays to hold onto the wine longer before selling it, because the value of the wine is increasing at the same rate as before. Again, you should buy all the cases.
10. Reexamine the capital investment decision in the disposable diaper industry (Example 15.3) from the point of view of an incumbent firm. If P&G or Kimberly-Clark were to expand capacity by building three new plants, they would not need to spend $60 million on R&D before start-up. How does this advantage affect the NPV calculations in Table 15.5? Is the investment profitable at a discount rate of 12 percent?

If the only change in the cash flow for an incumbent firm is the absence of a $60 million expenditure in the present value, then the NPV calculations in Table 15.5 simply increase by $60 million for each discount rate:

<table>
<thead>
<tr>
<th>Discount Rate:</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>140.50</td>
<td>43.50</td>
<td>-15.10</td>
</tr>
</tbody>
</table>

To determine whether the investment is profitable at a discount rate of 12 percent, we must recalculate the expression for NPV. At 12 percent,

$$NPV = -60 - \frac{93.4}{(1.12)} - \frac{56.6}{(1.12)^2} + \frac{40}{(1.12)^3} + \frac{40}{(1.12)^4} + \frac{40}{(1.12)^5} + \frac{40}{(1.12)^6} + \frac{40}{(1.12)^7} + \frac{40}{(1.12)^8} + \frac{40}{(1.12)^9} + \frac{40}{(1.12)^{10}} + \frac{40}{(1.12)^{11}} + \frac{40}{(1.12)^{12}} + \frac{40}{(1.12)^{13}} + \frac{40}{(1.12)^{14}} + \frac{40}{(1.12)^{15}} = $$

$16.3$ million.

Thus, the incumbent would find it profitable to expand capacity.

11. Suppose you can buy a new Toyota Corolla for $20,000 and sell it for $12,000 after six years. Alternatively, you can lease the car for $300 per month for three years and return it at the end of the three years. For simplification, assume that lease payments are made yearly instead of monthly, i.e., are $3,600 per year for each of three years.

a. If the interest rate, $r$, is 4 percent, is it better to lease or buy the car?

To answer this question, you need to compute the NPV of each option. The NPV of buying the car is:

$$-20,000 + \frac{12,000}{1.04^6} = -10,516.22.$$ 

The NPV of leasing the car is:

$$-3,600 - \frac{3,600}{1.04} - \frac{3,600}{1.04^2} = -10,389.94.$$ 

In this case, you are better off leasing the car because the NPV is higher.

b. Which is better if the interest rate is 12%?

To answer this question, you need to compute the NPV of each option. The NPV of buying the car is:

$$-20,000 + \frac{12,000}{1.12^6} = -13,920.43.$$ 

The NPV of leasing the car is:

$$-3,600 - \frac{3,600}{1.12} - \frac{3,600}{1.12^2} = -9,684.18.$$ 

In this case, you are better off leasing the car because the NPV is higher.
c. At what interest rate would you be indifferent between buying and leasing the car?

You are indifferent between buying and leasing if the two NPV's are equal or:

\[
-20,000 + \frac{12,000}{(1 + r)^6} = -3,600 - \frac{3,600}{(1 + r)} - \frac{3,600}{(1 + r)^2}.
\]

In this case, you need to solve for \( r \). The easiest way to do this is to use a spreadsheet and calculate the two NPV’s for different values of \( r \). Observe first that the interest rate will be something less than 4% given that at 4% the best option was to lease, and as the interest rate rose to 12% leasing became even a better option. The interest rate will be in the neighborhood of 3.8%.

<table>
<thead>
<tr>
<th>( r )</th>
<th>NPV Buy</th>
<th>NPV Lease</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>-9,950.19</td>
<td>-10,488.49</td>
</tr>
<tr>
<td>0.035</td>
<td>-10,237.99</td>
<td>-10,438.90</td>
</tr>
<tr>
<td>0.037</td>
<td>-10,350.41</td>
<td>-10,419.24</td>
</tr>
<tr>
<td>0.038</td>
<td>-10,406.06</td>
<td>-10,409.44</td>
</tr>
<tr>
<td>0.04</td>
<td>-10,516.22</td>
<td>-10,389.94</td>
</tr>
</tbody>
</table>

12. A consumer faces the following decision: she can buy a computer for $1,000 and pay $10 per month for Internet access for three years, or she can receive a $400 rebate on the computer (so that it costs $600) but agree to pay $25 per month for three years for Internet access. For simplification, assume that the consumer pays the access fees yearly (i.e., $10 per month = $120 per year).

a. What should the consumer do if the interest rate is 3 percent?

To figure out the best option, you need to calculate the NPV in each case. The NPV of the first option is:

\[
-1,000 - 120 - \frac{120}{1.03} - \frac{120}{1.03^2} = -1,349.62.
\]

The NPV of the second option with the rebate is:

\[
-600 - 300 - \frac{300}{1.03} - \frac{300}{1.03^2} = -1,474.04.
\]

In this case, the first option gives a higher NPV so the consumer should pay the $1000 now and pay $10 per month for Internet access.

b. What if the interest rate is 17 percent?

To figure out the best option, you need to calculate the NPV in each case. The NPV of the first option is:

\[
-1,000 - 120 - \frac{120}{1.17} - \frac{120}{1.17^2} = -1,310.23.
\]

The NPV of the second option with the rebate is:

\[
-600 - 300 - \frac{300}{1.17} - \frac{300}{1.17^2} = -1,375.56.
\]

In this case, the first option gives a higher NPV so the consumer should pay the $1000 now and pay $10 per month for Internet access.
c. At what interest rate is the consumer indifferent between the two options?

The consumer is indifferent between the two options if the NPV of each option is the same. To find this interest rate set the NPV's equal and solve for r.

\[-1,000 - 120 - \frac{120}{1 + r} - \frac{120}{(1 + r)^2} = -600 - 300 - \frac{300}{1 + r} - \frac{300}{(1 + r)^2}\]

\[220 = \frac{180}{1 + r} + \frac{180}{(1 + r)^2}\]

\[220(1 + r)^2 = 180(1 + r) + 180\]

\[220r^2 + 260r - 140 = 0.\]

Using the quadratic formula to solve for the interest rate r results in r≈40.2% (approximately).