An Iterative Approach for Applying Multiple Currents to a Body Using Voltage Sources in Electrical Impedance Tomography.

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Abstract—An approach for using voltage sources to produce a desired current pattern in an ACT-type EIT system is presented. An iterative adaptive algorithm generates the necessary voltage pattern that will result in the desired current pattern. The convergence of the algorithm is shown under the condition that the estimation error of the linear mapping from voltage to current is small. The simulation results are presented along with the implication of the convergence condition.

Keywords—Electrical impedance tomography, multiple currents, iterative algorithm, voltage source, ACT, EIT

I. INTRODUCTION

Electrical impedance tomography (EIT) is a technique for determining the electrical conductivity and permittivity distribution within the interior of a body from measurements made on its surface. Typically, currents are applied through electrodes placed on the body’s surface and the resulting voltages are measured. Alternately, voltages can be applied and the resulting currents are measured. Recent reports on a number of EIT systems can be found in [3]-[7]. Some systems apply currents to a pair of adjacent electrodes, with the current entering at one electrode and leaving at another, and measure voltages on the remaining electrodes. In these Applied Potential Tomography (APT) systems, the current is applied to different pairs of electrodes, sequentially to produce enough data for an image. In Adaptive Current Tomography (ACT) systems, currents are applied to all the electrodes simultaneously and multiple patterns of currents are applied to produce the data necessary for an image. If the body being imaged is circular or cylindrical and measurements are performed using a single ring of electrodes around the body, the most common current patterns are spatial sinusoids of various frequencies. In this paper, we focus on a current delivery system for an ACT-type EIT system that uses voltage sources.\textsuperscript{1}

The image reconstruction problem in EIT is ill-posed, and large changes in the conductivity and permittivity in the interior can produce small changes in the currents or voltages at the surface. As a result, measurement precision in EIT systems is of critical importance. It is known that when current is applied and the resulting voltages are measured, the errors in the measured data are reduced as the spatial frequency increases, proportional to the inverse of the spatial frequency. Conversely, the error is amplified in proportion to the spatial frequency when a voltage distribution is applied and the resulting current is measured [1]. Hence, the current source mode is superior to the voltage mode in terms of the high frequency noise suppression and higher accuracy in the conductivity image.

In practice, however, current sources are difficult as well as expensive to build [2]. Building a high precision current source is a technologically challenging task. The current source must have output impedance sufficiently large compared to the load, at the operating signal frequency to ensure that the desired current is applied for various loads. It is even more difficult to design a current source if the EIT system is to operate over a wide range of signal frequencies, as is required for EIT spectroscopy. The implementation of high-precision current sources has generally required the use of calibration and trimming circuits to adjust output impedance up to sufficient levels, yielding relatively complex circuits.

A voltage source, however, is easier and less expensive to build and operate compared to a current source. It requires smaller circuit board space, and can be easily and quickly calibrated. EIT systems using voltage sources have been implemented, though these systems suffer from increased sensitivity to the high frequency noise described above. Ideally, one would like the simplicity of voltage sources with the noise advantages of current sources.

Here we present an approach for using voltage sources to produce the desired current pattern in an ACT-type EIT system. The amplitude and phase of a voltage source need to be adjusted in a way that produces the desired current. An iterative algorithm was reported in [8] where the individual voltage sources are adjusted using a concept of an effective load. The current was shown to converge to the desired value in majority of the experiments. This paper presents a computation algorithm that generates the voltages in a more systematic way, and the condition of the current convergence is given in an explicit form. At present, the EIT system in Rensselaer Polytechnic Institute is ACT 3, which uses current sources only. The next version of EIT system under development is ACT 4 and it has voltage as well as the current sources. This work is an effort to replace the high precision current source by generating the current by software using the voltage source.

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Let \( I = (I_1, I_2, \cdots, I_k)^T \) denote an \( L \times L \) electrode current vector where \( I_k \) is the current value on electrode \( n \), and \( L \) is the number of electrodes. Similarly let \( V = (V_1, V_2, \cdots, V_k)^T \) denote an \( L \times L \) electrode voltage vector. The mapping from the applied electrode voltage \( V \) to the measured electrode current \( I \) can be represented using a constant \( L \times L \) matrix \( A \), so that \( I = AV \), provided that the change with time in the electrical conductivity of human body under examination is assumed to be negligible or the change is slow compared to the fast sampling time of the measurement data. Since the magnitude and the phase of the currents and voltages are used in the conductivity and permittivity reconstruction, the elements of \( I, V \) and \( A \) are complex numbers. The goal is to compute voltage \( V^* \) that will generate the desired electrode current pattern \( I^* \). The exact value of \( A \) can not be determined. The estimate of \( A \), denoted as \( \hat{A} \), can be obtained experimentally by applying a set of independent current patterns and measuring the corresponding output voltages. Then, \( \hat{A} \) can be used to compute \( V^* \). However, \( \hat{A} \) would contain errors due to modeling errors in the geometry of the electrodes in addition to the measurement errors. In this paper, an iterative algorithm for computing the voltage \( V^* = (V_1^*, V_2^*, \cdots, V_k^*)^T \) is presented that will produce a desired current pattern \( I^* \) with high precision in the presence of the estimation errors in \( \hat{A} \). Let us consider the following algorithm.

**Algorithm 1.** Given a nonsingularity estimate \( \hat{A} \) of the linear mapping \( A \) from voltage to current, \( I = AV \), a desired current pattern \( I^* \), and error tolerance \( \varepsilon \), find the voltage \( V^* \) that will produce \( I^* = AV^* \) such that \( \| e \| = \| I^* - I^* \| < \varepsilon \).

1. \( e_0 = I^* \), \( V^0 = 0 \), \( k = 1 \)
2. Compute \( V^k = V^{k-1} + \hat{A}^{-1}e_{k-1} \)
   - Apply \( V^k \), and measure \( I^k \). Compute \( e_k = I^k - I^k \).
3. If \( \| e_k \| < \varepsilon \) then \( V^* = V^k \) and stop. Else go to 2

**Theorem 1.** The \( k \)-th error in Algorithm 1 is \( e_k = Q^k I^k \) where \( Q = (1 - \hat{A} \hat{A}^{-1}) \). Furthermore, if \( \| Q \| < 1 \), then \( \| e_k \| < \| e_{k-1} \| \) and \( \| e_k \| < \| Q \| \| e_{k-1} \| \) hold for \( k \geq 1 \).

Let us suppose the assumption is true for \( (k-1) \)-th step, i.e. \( e_{k-1} = Q^{k-1} I^k \). Then, \( V^k = V^{k-1} + \hat{A}^{-1} e_{k-1} = V^{k-1} + \hat{A}^{-1} (I - \hat{A} \hat{A}^{-1}) Q^{k-1} I^k \). Also, \( I^k = AV^k = AV^{k-1} + \hat{A}^{-1} (I - \hat{A} \hat{A}^{-1}) Q^{k-1} I^k \) \( = I^{k-1} + \hat{A}^{-1} (I - \hat{A} \hat{A}^{-1}) Q^{k-1} I^k \)

The error at \( k \)-th step is \( e_k = I^k - I^* = I^{k-1} - \hat{A}^{-1} (I - \hat{A} \hat{A}^{-1}) Q^{k-1} I^k \)

Thus, the error expression is proved. Next, the convergence of the error is shown.

**Theorem 2.** Let \( e_0 = I^* \) and \( Q = (1 - \hat{A} \hat{A}^{-1}) \). Also, \( \| e_0 \| = \| Q^0 I^* \| \leq \| Q \| \| e_0 \| \leq \| Q \| \| e_1 \| \leq \cdots \leq \| Q \| \| e_k \| \leq \| Q \| \| e_{k-1} \| \leq \| e_{k-1} \| \)

Since \( \| Q \| < 1 \) by assumption, we have \( \| e_k \| < \| Q \| \| e_{k-1} \| \)

Theorem 1 requires the nonsingularity of \( \hat{A} \) as well as the bound on the estimation error of \( \hat{A} \) in the form of \( \| Q \| < 1 \). When the voltage pattern is applied and a current pattern is produced, the sum of the electrode currents through the body is zero. Because of this constraint on the electrode current values, the dimension of the current vector space is \( L - 1 \), while the dimension of the voltage space is \( L \). The linear mapping \( A \) from the voltage space to the current space given by \( I = AV \) is a singular mapping and it can not be used in Theorem 1 directly.

The linear mapping from voltage space to the current space can be formulated as a nonsingular mapping if the sum of the applied electrode voltages is constrained to be zero. Then, the dimensions of the voltage subspace and current subspaces are both \( L - 1 \), and the mapping from \( L - 1 \) dimensional voltage subspace to \( L - 1 \) dimensional current subspace can be represented by a \((L-1) \times (L-1)\) nonsingular matrix. Let us choose an orthonormal basis set \( \{ T^1, T^2, \cdots, T^{L-1} \} \) for the voltage and current subspaces, such that \( \sum_{n=1}^{L-1} I_n = \sum_{n=1}^{L-1} V_n = 0 \), \( T^k = [T^k_1 \cdots T^k_{L-1}]^T \), \( < T^k, T^\ell > = \delta_{k\ell} \), \( \sum_{n=1}^{L-1} T^k_n = 0 \), where \( < T^k, T^\ell > \) is the inner product of \( T^k \) with \( T^\ell \). The current and voltage vectors can be represented as coordinate vectors with respect to the basis vector set.

\[
I = \sum_{n=1}^{L-1} i_n T^n, \quad V = \sum_{n=1}^{L-1} v_n T^n
\]

In the above expression, \( i_n \) and \( v_n \) are the \( n \)-th coordinates of the current \( I \) and voltage \( V \) with respect to the basis \( T^n \). Apply voltage \( T^k \) and measure \( I^k \). Then, \( I^k = AT^k \). The relationship from the applied voltage \( V \) to the measured current \( I \) is

\[
I = AV
\]

\[
\sum_{n=1}^{L-1} I_n T^n = \sum_{n=1}^{L-1} V_n AT^n
\]

Taking the inner product of both sides with \( T^u \), \( u = 1, 2, \cdots, L - 1 \)

\[
i_u = \sum_{n=1}^{L-1} < T^u, T^n > v_n, \quad u = 1, 2, \cdots, L - 1
\]

Let \( i = i_1, i_2, \cdots, i_{L-1} \), \( v = [v_1, v_2, \cdots, v_{L-1}] \), then

\[
i = i_1, i_2, \cdots, i_{L-1}
\]

\[
v = [v_1, v_2, \cdots, v_{L-1}]
\]

Then, the linear mapping from the coordinate vector \( v \) to the coordinate vector \( i \) is nonsingular, and described by...
where \( B \) is a \((L-1)\times (L-1)\) nonsingular matrix.

\[
B = \begin{bmatrix}
< T^1, I_1^1 > & \cdots & < T^1, I_1^{L-1} > \\
< T^2, I_1^1 > & \cdots & < T^2, I_1^{L-1} > \\
\vdots & \ddots & \vdots \\
< T^{L-1}, I_1^1 > & \cdots & < T^{L-1}, I_1^{L-1} > \\
\end{bmatrix}
\]

(2)

Algorithm 2. Given a desired current \( I^d \), a basis set \( \{ T^i \}_{i=1}^{L-1} \), and the relationship from voltage coordinate vector to current coordinate vector \( i = Bv \), find an estimate of \( i \), used in Algorithm 1.

1. Apply voltage \( T^1 \) and measure \( I^1 \), \( k=1, \ldots, L-1 \).
2. Compute \( \hat{B} \) from (2).
3. Compute \( i^d = \begin{bmatrix} i^d_1 \\ i^d_2 \\ \vdots \\ i^d_{L-1} \end{bmatrix} = \begin{bmatrix} < I^1, T^1 > \\ < I^1, T^2 > \\ \vdots \\ < I^{L-1}, T^{L-1} > \end{bmatrix} \)

Now, we can use the nonsingular mapping \( B, i = Bv \), in Algorithm 1, and the procedure is summarized below.

Algorithm 3. Given a desired current \( I^d \), an error tolerance \( \varepsilon \), find the voltage \( V^* \) that will result in the current \( I^d \) such that \( \|\varepsilon\| = \| I^d - I^* \| < \varepsilon \).

1. Using Algorithm 2, compute \( \hat{B}, i^d \)
2. Let \( e_0 = i^d, V_0 = V_0^0, k = 1 \)
3. Compute \( V^k = x^k + \hat{B} e^k \).
4. Apply \( V^k = \sum_{j=1}^{L-1} V^k_j T^j \), and measure \( I^k \).
5. Compute \( i^k = \begin{bmatrix} i^k_1 \\ i^k_2 \\ \vdots \\ i^k_{L-1} \end{bmatrix} = \begin{bmatrix} < I^1, T^1 > \\ < I^2, T^2 > \\ \vdots \\ < I^{L-1}, T^{L-1} > \end{bmatrix} \)
6. Compute \( e_k = i^d - i^k \)
7. If \( \| e_k \| < \varepsilon \) then \( V^* = V^k \) and stop. Else go to 3.

Fig. 1. Convergence of the current output when no current measurement noise is present. X axis represents iteration counts.

In order to simulate the estimation error, random multiplicative errors and additive errors were added to each element of \( B \) to make up \( \hat{B} \). For example, to introduce 1% multiplicative error, a random number \( x \) was generated with uniform distribution between -0.01 and +0.01, and \( (1+x) \) was multiplied to each element of \( B \). For additive error, \( xB_{max} \) was added to each element of \( B \), where \( B_{max} \) is the element of \( B \) with maximum absolute value. In order to simulate the current measurement noise, a set of random numbers was generated with uniform distribution between -1 and 1, the magnitude were adjusted so that the SNR is 105dB as reported in [3], and were added to the currents.

The desired current value used in the simulation was \( I^d_i = 0.2 \cos \theta_i + j0.1 \sin \theta_i \) (mA) for the \( k \)-th electrode. The real part of \( I^d \) is one of the actual current patterns used in the ACT 3 measurements. The imaginary part was added for test purposes. Fig.1 shows the convergence of the current as the iteration count increases. Five lines represent the results with different multiplicative and additive errors. For example, error 1.0% means that the multiplicative error of 1% and additive error of 1% were introduced as the estimation error. The lower figure shows the magnified portion of the upper figure. Note that the resolution of the 16 bit ADC is \( 1/2^{16} = 1.5x10^{-5} \) and the errors decrease below this value after 5 ~ 12 iterations as shown in Fig.1.

Also note that Theorem 1 implies that if the initial error \( \| e_0 \|_2 = 1 \) and \( \| Q \| \leq 0.1 \), it will take at most \( k=5 \) iterations to reduce the error below the resolution of the 16 bit ADC. It can be seen that when the estimation errors are 1.5%, 2.0%, and 2.5%, \( \| Q \| \) are greater than 1, but the current still converges to the desired value. This is because the convergence condition \( \| Q \| < 1 \) is a sufficient condition. Even when it is not satisfied, the current convergence is still
possible, though not guaranteed. Fig. 2 shows the same simulation with the current measurement error added. It is seen that the current almost converges to the desired value, within the error bounds set by the noise. The remaining error is the consequence of the measurement noise.

![Convergence of current (estimation error in all elements of B)](image)

Fig. 2. Convergence of the current output when current measurement noise is present. X axis represents iteration counts.

The speed of convergence and whether the current will converge at all depend on the magnitude of the estimation error in the form of $\|Q\| = \|I - BB^{-1}\|$. If $\|Q\| < 1$, it is guaranteed to converge to the desired value by Theorem 1. The speed of the convergence depends on the magnitude of $\|Q\|$. If $\|Q\| > 1$, current may still converge as shown in Fig. 1 and 2. The next question is how realistic the condition $\|Q\| < 1$ is in practice. Fig. 3 shows the behavior of $\|Q\|$ with the variation of multiplicative and additive errors. Multiplicative error and additive errors were varied independently, and their effect on $\|Q\|$ was studied. Since the errors were generated by random numbers, for each combination of multiplicative error and additive error, $\|Q\|$ was computed 1000 times and the maximum value was used as the value of $\|Q\|$. It can be seen from the upper figure that $\|Q\| < 1$ when additive error was less than 1%. The multiplicative error had less significant influence because $B$ was a diagonal matrix and the off-diagonal elements were zero. Since $B$ is diagonal, we can force the off-diagonal elements of $B$ to be zero, and apply estimation errors to the diagonal elements only. In this case, it can be seen from the lower figure that $\|Q\| < 1$ when additive error was less than 2.5%. This suggests that the knowledge of the true form of $B$ can be used to reduce the effect of the estimation error.

### IV. Discussion and Conclusions

It was shown that if the linear mapping from the voltage coordinate vector to the current coordinate vector can be estimated within a certain error bound, the current output produced by applying the voltage can be made to approach the desired value asymptotically. It was seen that when the convergence condition $\|Q\| < 1$ was satisfied, the current output approached the desired value. Additive error of 2.5% with multiplicative error of 7% could be tolerated to maintain the condition $\|Q\| < 1$. In practice, however, since we cannot know the true value of $B$ but only have the estimate $\hat{B}$, it is not possible to determine the value of $\|Q\|$. If the current converges to a value, it is an indirect indication that the condition $\|Q\| < 1$ may have been satisfied. Experimental verification needs to be done for this algorithm, and the effect of the measurement error on the magnitude of $\|Q\|$ should be studied in the future.

### References